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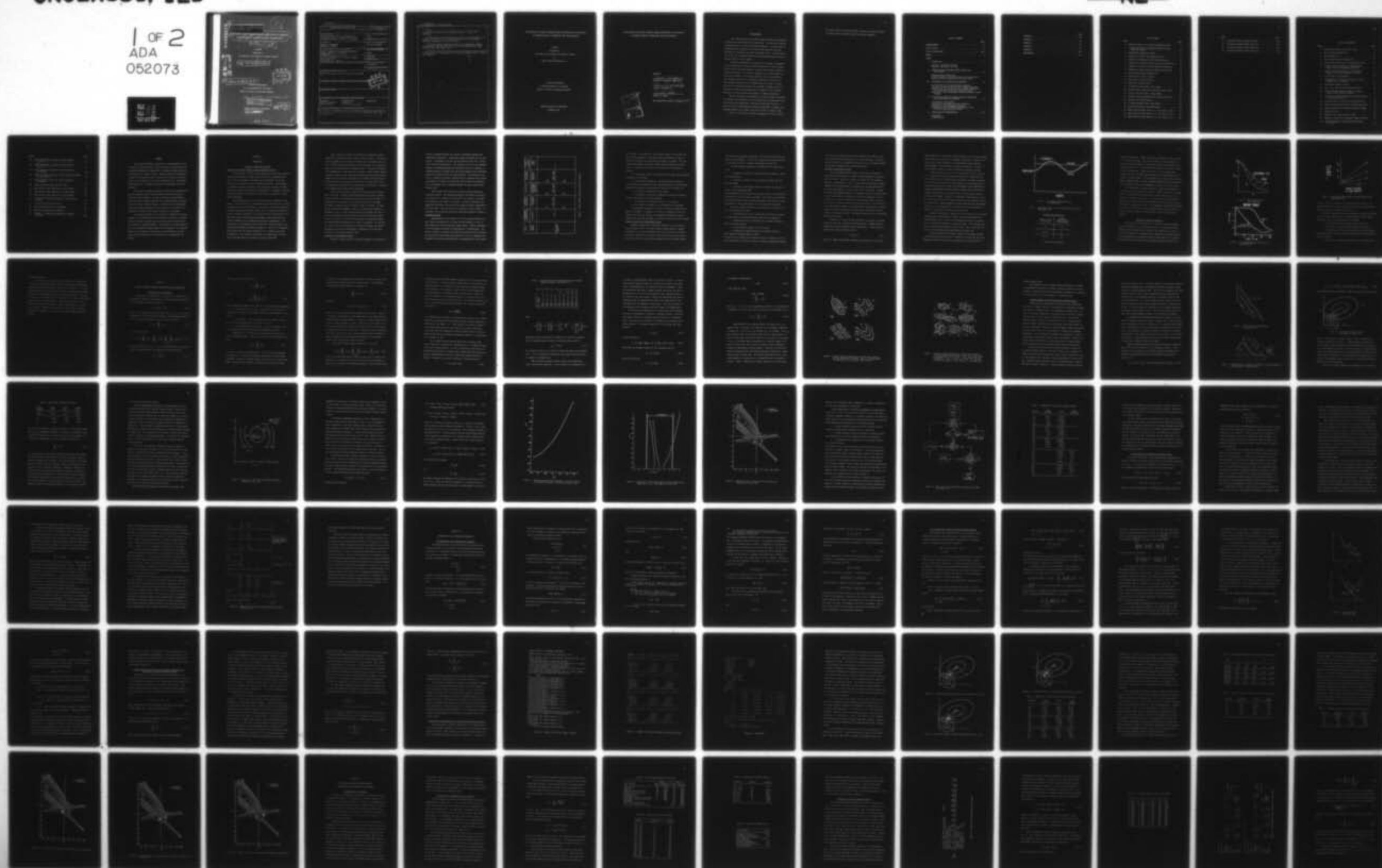

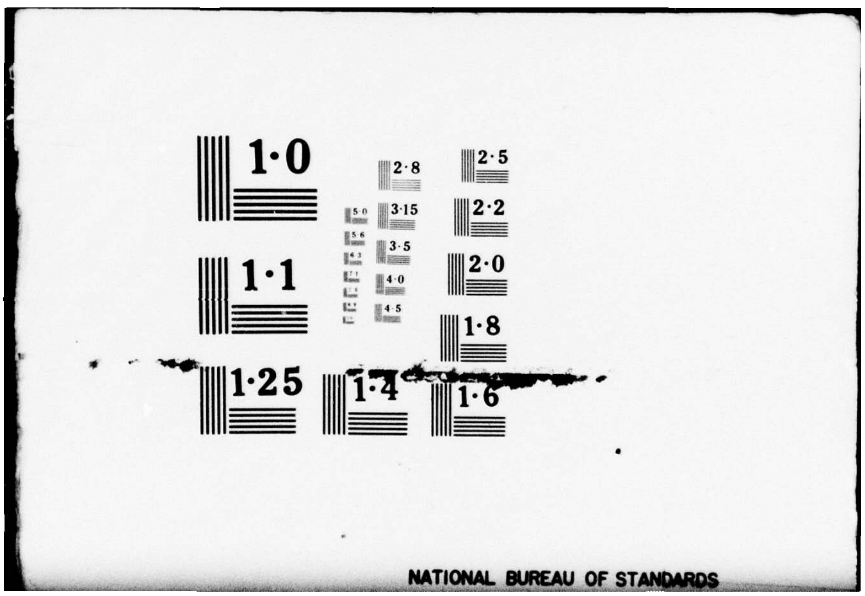
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AN APPLICATION OF MULTIPLE RESPONSE SURFACE OPTIMIZATION TO THE ANALYSIS OF TRAINING EFFECTS IN OPERATIONAL TEST AND EVALUATION.

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Presented to

The Faculty of the Division of Graduate Studies

By

(10) Vernon Manuel Bettencourt, Jr.

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This report is a summary report of four studies in support of the application of statistical theory to design and evaluation of operational tests. The four topics are:

a. "A Methodology for Determining the Power of MANOVA when the Observations are Serially Correlated," by Norviel R. Eyrich, CPT, Artillery.

b. "An Application of Multiple Response Surface Optimization to the Analysis of Training Effects in Operational Test and Evaluation," by Vernon M. Bettencourt, Jr., CPT, Artillery.

c. "A Cost Optimal Approach to Selection of Experimental Designs for Operational Testing under Conditions of Constrained Sample Size" by Sam W. Russ, MAJ, Signal Corps.

d. "An Application of Bayesian Statistical Methods in the Determination of Sample Size for Operational Testing in the US Army," by Robert M. Baker, CPT, Infantry.

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AN APPLICATION OF MULTIPLE RESPONSE SURFACE OPTIMIZATION TO THE ANALYSIS OF TRAINING EFFECTS IN OPERATIONAL TEST AND EVALUATION

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TABLE OF CONTENTS

		Page
ACKNOWLEDGMENTS		ii
LIST OF TABLES		vi
LIST OF ILLUSTRATIONS		viii
SUMMARY		x
Chapter		
I.	INTRODUCTION	1
	Overview: Operational Testing Objective, Procedure, and Scope	
II.	REVIEW OF MULTIPLE RESPONSE SURFACE THEORY AND OPTIMIZATION	14
	Response Surface Methodology Multiple Response Surface Optimization Literature Survey Multiple Objective Optimization Literature Survey	
III.	DEVELOPMENT OF AN OPTIMIZATION METHODOLOGY	45
	The Frank-Wolfe Linear Approximation Algorithm The Geoffrion-Dyer Interactive Vector Maximal Algorithm Adaptation of the Interactive Vector Maximal Algorithm to the Optimization of Multiple Response Surfaces Application of the Methodology to Multiple Response Surface Problems	
IV.	APPLICATION OF MULTIPLE RESPONSE SURFACE OPTIMIZATION TO AN OPERATIONAL TEST PROBLEM	72
	Introduction to the Problem Utilization of the AMSAA Tank Duel Simulation Derivation of Multiple Response Surfaces Application of the Optimization Methodology to the Derived Multiple Response Surfaces	
V.	CONCLUSIONS AND RECOMMENDATIONS	107
	Conclusions Recommendations	

	Page
APPENDIX A	109
APPENDIX B	112
APPENDIX C	121
APPENDIX D	124
APPENDIX E	134
BIBLIOGRAPHY	137

LIST OF TABLES

Table		Page
1.	Quantified Effect of the Learning-Forgetting Curve	9
2.	Uniform Precision and Orthogonal Rotatable Central Composite Designs	18
3.	Umland-Smith Optimization Results	27
4.	Comparison of Fields' and Original Results	37
5.	Comparison of Umland and Smith Problem Solutions	65
6.	Algorithm Search Results, Myers and Carter Problems One .	67
7.	Comparison of Myers and Carter Problem One Soltuions . . .	67
8.	Comparison of Myers and Carter Problem Two Solutions . . .	68
9.	Input Variable Normal Distributions	75
10.	Distribution of U(0,1) Deviates	75
11.	Distribution of N(0,1) Deviates	76
12.	Fixed Input Variable Values	76
13.	2^3 Design Variable Values, First Design	80
14.	Steepest Ascent Optimization From First Center Point . . .	843
15.	2^3 Design Variable Values, Second Design	84
16.	Steepest Ascent Optimization From Second Center Point . .	87
17.	Uni-Direction Search Along ξ_3	87
18.	2^3 Design Variable Values, Third Design	89
19.	Axial Points Added to the Third Design	89
20.	AMSAA Tank Duel Model Output at $\xi_1 = 23.0$ and $\xi_2 = 18.0$.	99
21.	AMSAA Tank Duel Model Output at $\xi_1 = 8.0$ and $\xi_2 = 5.0$ . .	99
22.	AMSAA Tank Duel Model Output at $\xi_1 = 10.7$ and $\xi_1 = 8.2$ . .	102

Table		Page
23.	Sensitivity Analysis Toward (8.0,5.0)	103
24.	Sensitivity Analysis Toward (8.0,15.0)	104
25.	Sensitivity Analysis Toward (16.0,5.0)	105
26.	Sensitivity Analysis Toward (16.0,15.0)	106

LIST OF ILLUSTRATIONS

Figure		Page
1.	Major Defense Systems Acquisition Process	4
2.	The Learning-Forgetting Curve	9
3.	The Performance Gap	11
4.	The LAW Weapons Test Performance Gap.	11
5.	System Total Combat Power at Varying Effectiveness . .	12
6.	Response Surfaces Generated by a Second Degree Equation With Two Independent Variables	21
7.	Response Surfaces Generated by a Second Degree Equation with Three Independent Variables	22
8.	Yield Planes of Box Experiment	25
9.	Superimposition of Constraint Response on Primary Response in Box Experiment	25
10.	Umland-Smith Response Surfaces	26
11.	Lind, et al, Cost and Yield Response Contours	29
12.	Maximum Estimated Primary Response at Specific Values of the Constraint Response	32
13.	Conditions of Constrained Maxima on Primary Response for Fixed Values of $\hat{y}_s$	33
14.	Response Surface of Myers and Carter Problem Two . . .	34
15.	Hooke and Jeeves Pattern Search Technique Flow Chart .	36
16.	Sample Output From the VM((Vector Maximal) Program . .	43
17.	Estimation of w_i^k	54
18.	Example of Data Input Computer Program	60
19.	Example of Interactive Optimization Computer Program .	61
20.	Algorithm Movement on Umland and Smith Problem $\hat{y}_s \leq 90.0$	64

Figure Page

21. Algorithm Movement on Umland and Smith Problem, $\hat{y}_s \leq 92.5$ 64

22. Algorithm Movement on Umland and Smith Problem, $\hat{y}_s \leq 95.0$ 65

23. First Iteration of Myers and Carter Problem Two Optimization 69

24. Second and Third Iterations of Myers and Carter Problem Two Optimization 70

25. Final Iteration of Myers and Carter Problem Two Optimization 71

26. Sample Output From AMSAA Tank Duel Model 78

27. SPSS Multiple Linear Regression of First Design 81

28. SPSS Multiple Linear Regression of Second Design 85

29. SPSS Multiple Linear Regression of Third Design 90

30. SPSS Multiple Polynomial Regression of Second Order Design 91

31. Derived Multiple Response Surface 94

32. Convergence Movement of the Methodology 97

33. Validation of Methodology Convergence 98

34. Movement of Optimization Methodology in Training Problem 101

SUMMARY

This research considers the analysis of training effects in operational test and evaluation. Previous analysis of weapons system effectiveness highlights the importance of including training effects in any evaluation of a weapons system. Computer simulation is proposed as a method of extending the scope of operational testing into areas for which it is not feasible to test in an operational test. The mutually supporting nature of computer simulations and operational tests are discussed.

Utilization of computer simulation facilitates the derivation of multiple response surfaces relating weapons system effectiveness to training related variables. The research adapts the Geoffrion-Dyer Interactive Vector Maximal algorithm into a methodology for the optimization of multiple response surfaces. Application of the methodology to multiple response problems previously solved in the literature is performed with results which compare favorably to the original.

A hypothetical analysis of the effects of training on the effectiveness of a new main battle tank is described in detail. The methodology is utilized to optimize four objective response functions which are functions of training variables. Utilization of the methodology results in an improved training program for test personnel, in a detailed analysis of the effects of training on the effectiveness of the new tank, and in the inclusion of this analysis in the operational test reports.

CHPATER I

INTRODUCTION

Overview: Operational Testing

Structure of the Major Defense Systems Acquisition Process

The large sums of federal moneys expended on major defense systems acquisition necessitate a highly structured and well safeguarded procedure. Both the Department of Defense and the Department of the Army utilize such a procedure in their acquisition processes. The procedure is designed to insure acquisition of only those major systems for which a valid need exists within the defense establishment. Department of Defense directives document the acquisition process and its procedures in great detail (60,63,64).

The acquisition cycle of a major Army system is comprised of six phases. The first phase is a determination by the Army staff that a valid requirement exists for the addition of the system to the active inventory. A Required Operational Capability (ROC) report, containing a statement of need and conceptual approach, is approved and issued by Department of the Army (50). Next is the conceptual development phase during which the system's hardware is in an experimental prototype configuration. The third phase is the validation phase in which the system's hardware is in engineering development prototype configuration. Next is the development phase during which the system's hardware is in a production prototype configuration. The fifth and sixth phases are, respectively, full production and deployment of the system to tactical units (60).

After issuance of the ROC, the Secretary of Defense must grant approval for the system to move to each of the next phases. The decision options available to the Secretary of Defense are to terminate the system, to permit the system to proceed to the next phase, or to retain the system in its present phase for remedial action. To provide information and recommendations to the Secretary of Defense at these decision points, a permanent advisory body, the Defense Systems Acquisition Review Council (DSARC), has been created. Membership of the DSARC includes the Deputy Secretary of Defense and Assistant Secretaries of Defense within areas of responsibility pertinent to the system under consideration. A meeting of the DSARC preceeds each decision point (64)

There exists a parallel acquisition structure within the Department of the Army. The Army Systems Acquisition Review Council (ASARC) has been created as a permanent advisory body to provide the Army's recommendation at each phase of the acquisition process to the DSARC. The ASARC is chaired by the Vice Chief of Staff of the Army. Its membership includes the Commander of the U. S. Army Material Command, the Commander of the U. S. Army Training and Doctrine Command, the Chief of Research, Development, and Acquisition, and pertinent Assistant Secretaries of the Army. To fulfill the requirement of advising the DSARC, the ASARC schedules meetings prior to those of the DSARC. The principle of civilian control over the military is upheld throughout the systems acquisition cycle by the requirement of affirmation by the Secretary of Defense at each phase transition (60).

Testing in the Acquisition Process

Testing of a major system is conducted throughout the acquisition

process to determine whether the system is satisfying technical and operational requirements. Acquisition testing is divided into two categories: a Development Test (DT) and an Operational Test (OT). The DT and OT have diverse objectives. The objective of the DT is to determine whether the engineering design and development process is complete, to determine whether the design risks have been minimized, and to determine whether the system will meet its specifications. The objective of the OT is to estimate the system's military worth in comparison with competing systems, to estimate its operational effectiveness and suitability in its environment, and to determine whether the system required modification (60).

Three distinct DT's and OT's are usually conducted during the acquisition process. The scheduled meetings of the ASARC are preceded by a DT and an OT. Results of the DT and OT are reported to the ASARC for inclusion in the report to the DSARC. To provide additional safeguards and validation, the DT and OT are conducted totally independent of each other (60). Only the OT will be of interest in this research. Sequencing of the acquisition process is graphically depicted in Figure 1.

Operatinal Testing

Responsibility for the conduct of the OT's on major defense systems within the Department of the Army has been delegated to the U.S. Army Operational Test and Evaluation Agency (OTEA). OTEA is independent of the developing, procuring and using agencies or organizations. The mission of OTEA is to support the material acquisition and force development processes by exercising responsibility for all OT's, managing force development testing and experimentation, and managing joint user testing

PHASE	REQUIREMENT DETERMINATION	CONCEPTUAL DEVELOPMENT	VALIDATION	FULL SCALE DEVELOPMENT	PRODUCTION AND DEPLOYMENT
SYSTEM STATUS	CONCEPTUAL	EXPERIMENTAL PROTOTYPE	ENGINEERING DEVELOPMENT PROTOTYPE	PRODUCTION PROTOTYPE	FINAL MODEL
ACQUISITION PROCESS	ROC —	OT I ASARC I — DSARC I DT I	OT II ASARC II — DSARC II DT II	OT III ASARC III — DSARC III DT III	—

Figure 1. Major Defense Systems Acquisition Process

for the Army. In an effort to stress military usage of the tested system, the OT is conducted utilizing typical user/operators, crews, or units in as realistic an operational environment as possible. OT's are conducted throughout the world by several diverse testing and tactical units. The objective of the OT is to provide the data necessary to estimate:

1. The military utility, operational effectiveness, and operational suitability of the system.

2. The system's desirability, considering systems already in service (base-line systems) and other competing systems, and the system's operational advantages and disadvantages from the user's perspective.

3. The need for modification of the system.

4. The adequacy of doctrine, organization, operating techniques, tactics, and training for system deployment.

5. The adequacy of maintenance support for the system.

6. The system's performance in a countermeasures environment.

An independent evaluation of each OT is prepared by OTEA and submitted to the ASARC. An emphasis is placed on a comparison of the proposed system, base-line systems, and competing developmental systems. Feedback from the ASARC and DSARC is utilized to modify future OT's.(61,62).

Computer Simulation in Operational Testing

Computer simulation is finding wide application as a predictive and investigative tool. Most major defense systems undergo a computer simulation in a tactical environment both before and after the issuance of the ROC. Simulation can provide useful pre-test and post-test information for each OT. An important consideration is that computer simula-

tions and OT's are mutually supporting. OT's provide verified data inputs for the simulation. In return the simulation provides predictions of input data for OT's or further investigates OT output data.

Pre-test computer simulation can enhance the OT in three basic areas:

1. Examine the identified critical operational issues to assess their significance.

2. Develop or discover critical operational issues that have been overlooked.

3. Provided a sensitivity analysis to indicate the accuracy required of each measurement (50).

This information will be obtained at relatively little cost and with the utilization of no test troops or equipment. The OT will be initialized with useful information and critical operational issues will be verified or identified. Data requirements in the test plan will be refined.

Post-test computer simulation can contribute to the success of an OT in the following four areas:

1. Constraining the scope of operational field tests to manageable proportions by providing analytical means for test extension.

2. Extending the OT into areas which are currently infeasible (such as two-sided combat).

3. Corroborating the impact of the OT results.

4. Supplying much needed operational performance inputs to other agencies utilizing simulation (50).

OT results can be combined with simulation results to fulfill the stringent requirements of statistical design of experiment methodology analysis.

OT results can be utilized as input for simulations of combat in real time events, thereby eliminating rest or safety time lags. Simulation can be utilized as an independent evaluation of an OT, thereby providing an additional safeguard to the acquisition process.

Training in Operational Testing

The relationship between systems effectiveness and crew/unit training has recently began to receive increased emphasis in the Department of the Army. There are a variety of reasons for this increased interest. Establishment of the U. S. Army Training and Doctrine Command (TRADOC) has institutionalized the importance of training and doctrine by fixing responsibility at a high level of the Army command. Without the troop and equipment demands of a belligerent theater, the main mission of the Army transforms to training for the next belligerency. The ascending cost of systems combined with a federal budget squeeze necessitates increased combat effectiveness from fewer weapons. As previewed in the recent Mid-East conflict, the sophistication and lethality of weapons systems on either side dictates a rapid, deadly, and decisive first encounter in any future conflict. The results of these factors is increased interest in training.

TRADOC is, of course, the major proponent of training in the Army. Within the last year, operations research analysts at TRADOC have been examining training and weapons system effectiveness. A general model of systems effectiveness has been derived.

$$E = f(w,p,t) \tag{1.1}$$

where E is combat effectiveness expressed as a function of w the perfor-

mance capability of the system, p the proficiency of the crew/unit manning the system, and t the tactic or technique of employment. Various DT results, such as those obtained by the Army Material Systems Analysis Agency (AMSAA), can be utilized to measure and quantify w. Results of OT's conducted by OTEA, can also be utilized in determining w (59).

Some inconsistencies arise in the consideration of p in Equation 1.1. A Department of Defense directive states that, "Operational Test and Evaluation will be accomplished by operational and support personnel of the type and qualification of those expected to use and maintain the system when deployed".(50) Most OT's are conducted with troops/units selected to satisfy this directive and then trained either by the unit or Equipment Training Team in accordance with a training package prepared by OTEA and/or TRADOC. Training is accomplished at home station, at the test site, and at Military Occupational Specialty (MOS) producing schools if required (50). Having undergone such well supervised and concentrated training, it is not unreasonable to assume that the test personnel are atypical of Army users in proficiency on the system.

Another inconsistency in Equation 1.1 is the effect of the learning-forgetting curve on proficiency. Figure 2 depicts the influence of a training season, that is a period of concentrated training in a specific area, on proficiency followed by a forgetting slump. The training cycles of most tactical units approximate such a curve. Table 1 quantifies the effect of the forgetting curve among infantry trainees (59).

The weapons system effectiveness utilized by the ASARC and DSARC is that obtained from the DT and OT. Equation 1.1 states that the aforementioned variation in actual user proficiency will cause variation in

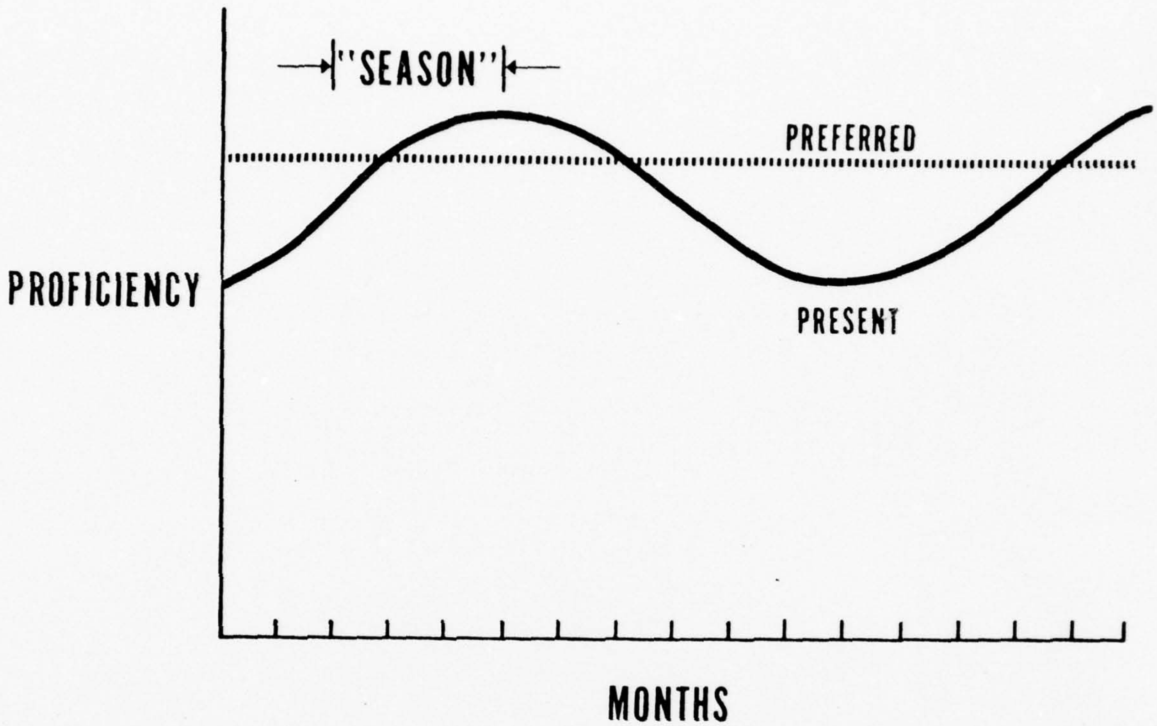

Figure 2. The Learning-Forgetting Curve.
From TRADOC (59).

Table 1. Quantified Effect of the Learning-Forgetting Curve.
From TRADOC (59)

Marksmanship Proficiency

NUMBER OF WEEKS IN THE ARMY	AVERAGE QUALIFICATION SCORE OBTAINED
4-5	52
14-16	44
24-52	*30

*1 point above unqualified

systems effectiveness. Figure 3 depicts the Probability of Hit and Kill of a system versus Range. Note the Performance Gap between AMSAA data (E_D) and actual performance in the hands of tactical troops (E_A) as predicted by Equation 1.1. This predicted Performance Gap has been verified in actual weapons test. In May 1974, the U. S. Army Infantry Board (USAIB) test fired the M72A2 Light Antitank Weapon (LAW) against moving targets at varying ranges. The Performance Gap uncovered by this test is shown in Figure 4 (59). The major problem encountered by the troops was a lack of proper training on the graduated lead sight for a moving target.

The implications of these variations in combat effectiveness for the national defense posture are profound. Figure 5 exhibits the varying levels of Systems Total Combat Power for a given inventory level N as a function of systems effectiveness. The effectiveness levels graphed are E_A the actual current level, E_D the designed effectiveness level, and E_M the optimum or maximum level (59). It is imperative that OTEA, functioning as a major source of data on weapons systems effectiveness to high level decision bodies, account for training levels in their OT reports and analysis.

Objective, Procedure, and Scope

The objective of this research is to develop an improved methodology for optimizing a set of operational test and evaluation performance measures which are functions of training. The research will consist of a review and adaptation of response surface methodology, multiple response surface optimization, and multiple objective optimization to the problem. The Geoffrion-Dyer Interactive Vector Maximal algorithm will then be re-

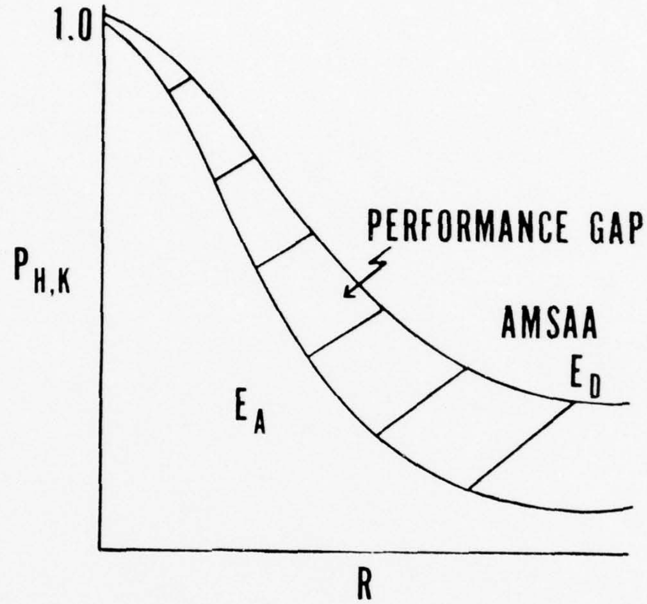

Figure 3. The Performance Gap.
From TRADOC (59).

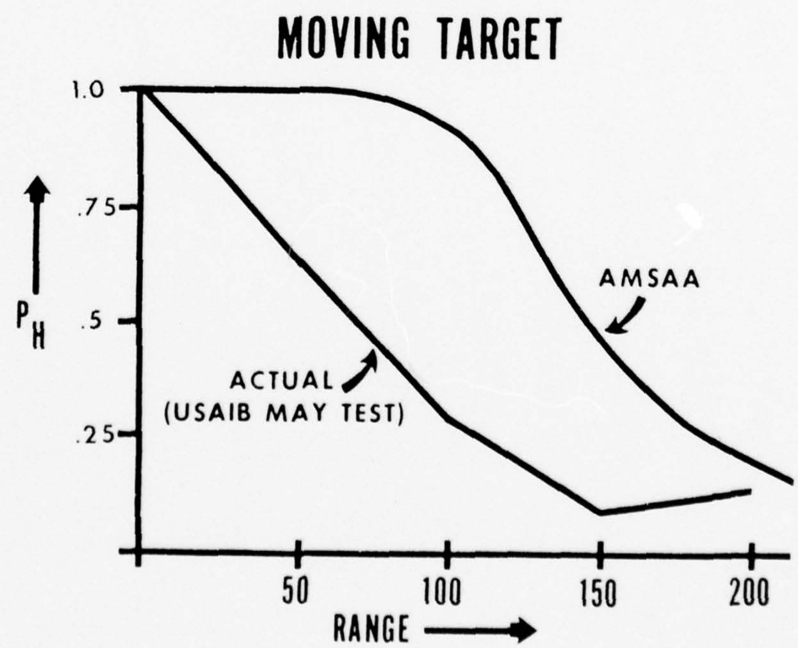

Figure 4. The LAW Weapons Test Performance Gap.
From TRADOC (59).

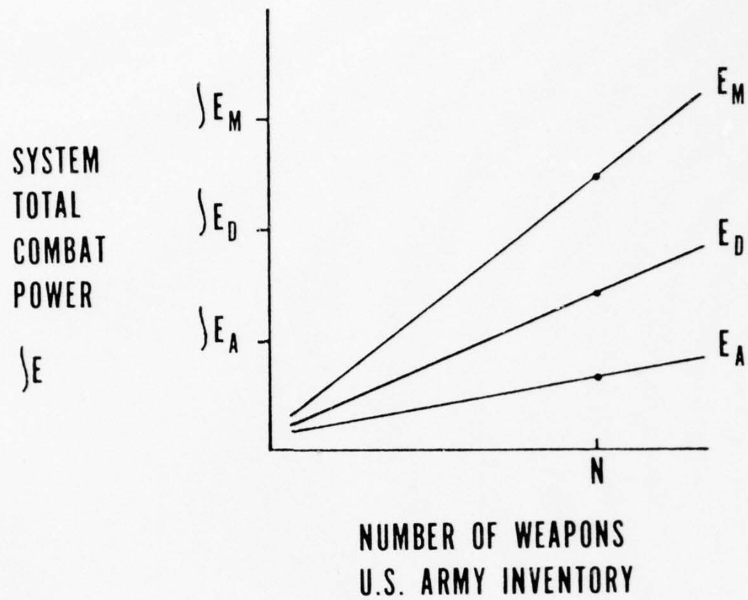

Figure 5. System Total Combat Power at Varying Effectiveness. From TRADOC (59).

viewed in detail and adapted to the multiple response problem. The adapted algorithm will then be applied to previously optimized multiple response surfaces to demonstrate its utility.

Multiple response surfaces and the adapted optimization algorithm will be related to OTEA by use of the AMSAA Tank Duel Model computer simulation. The military application will consider:

1. The extension of an OT through computer simulation.

2. The effect of training on tested system effectiveness.

3. The optimization of pre-test and tactical unit training programs concerning the tested system when confronted with multiple objectives or criteria.

4. The role of the military decision maker in the interactive

optimization process.

The scope of this research will be limited by four constraints. All data values utilized in this research are "best guess" hypothetical values which cannot necessarily be inferred to be realistic. For demonstration purposes, only one tactical scenario is analyzed with the AMSAA simualtion. The simulation is suited for various scenarios. The tactical scenario is two opposing tanks, in the open, at a range of 1000 meters, sighting each other simultaneously. Only mean time to fire the first round, mean time between subsequent rounds, and probability of sensing fired rounds are assumed to be functions of crew training. All other variables are assumed to be functions of the tested weapon system capabilities.

CHAPTER II

REVIEW OF MULTIPLE RESPONSE SURFACE THEORY AND OPTIMIZATION

Response Surface Methodology

Response surface methodology is a collection of statistical and mathematical techniques to approximate, utilizing designed experimentation, an unknown and complex function, say

$$\eta = f(\xi_1, \xi_2, \ldots, \xi_k) \tag{2.1}$$

where η is the dependent response variable and ξ_i, $i = 1,2,\ldots,k$, are the independent, controllable natural variables. The approximating model is usually a low order polynomial, such as a first order model

$$\eta = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \varepsilon \tag{2.2}$$

or a second order model

$$\eta = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i=1}^{k} \sum_{\substack{j=1 \\ i<j}}^{k} \beta_{ij} x_i x_j + \varepsilon \tag{2.3}$$

In these models the x_i, $i = 1,2,\ldots,k$, are design variables, coded within a region of experimentation for computational simplification by

$$x_{iu} = \frac{\xi_{iu} - \bar{\xi}_i}{S_i} \tag{2.4}$$

where ξ_{iu} is the u th level of ξ_i,

$$\bar{\xi}_i = \sum_{u=1}^{N} \xi_{iu}/N,$$

and

$$S_i^2 = \frac{\sum_{u=1}^{N} (\xi_{iu} - \bar{\xi}_i)^2}{N} \tag{2.5}$$

Three fundamental assumptions are involved in response surface methodology:

1. The structure $\eta = f(x_1, x_2, \ldots, x_k)$ exists and is either very complicated or unknown. The variables involved are quantitiative or continuous.

2. The function f can be approximated in the region of interest by a low order polynomial such as Equation 2.2 or 2.3.

3. The independent variables $x_1, x_2, \ldots, x_k$ are controlled in the data collection process and measured with negligible error (47).

Optimization of a response surface begins with a search for the region of maximum response. Initially a first order fitted response function,

$$\hat{y} = b_0 + \sum_{i=1}^{k} b_i x_i, \tag{2.6}$$

is fitted to a region of experimentation. This fitting is accomplished through the use of statistically designed experiments and least squares regression. Generally orthogonal designs are used to fit the first order model, since they greatly simplify computations and yield uncorrelated

estimates of the response model coefficients. Next the response is improved by moving along the path of steepest ascent. Using LaGrange Multipliers to maximize Equation 2.6 subject to

$$\sum_{i=1}^{k} x_i = R^2, \tag{2.7}$$

results in

$$x_j = b_j/2\mu \ (j = 1,2,\ldots,k) \tag{2.8}$$

where μ is a conveniently selected increment along the path. Equation 2.8 yields an initial point of experimentation for each design variable along the path of steepest ascent. A search is conducted along the path until an optimum response is reached. Addition of center points to the first order design at this improved point will permit a formal analysis of variance and a test for lack of fit. Should these reveal significant lack-of-fit for the first order fitted response function or should the path of steepest ascent yield minimal improvement, the experimenter usually fits a second order response function.

Second order fitted response functions are of the form

$$\hat{y} = b_0 + \sum_{i=1}^{k} b_i x_i + \sum_{\substack{i=1 \\ i<j}}^{k} \sum_{j=1}^{k} b_{ij} x_i x_j + \sum_{i=1}^{k} b_{ii} x_i^2 \tag{2.9}$$

There is a considerable amount of theory on the choice of design to fit Equation 2.9. Consideration is given to the bias of the predicted response or the variance of the predicted response. Uniform Precision and

Orthogonal Rotatable Central Composite Designs have received the greatest use in practice. A Central Composite Design (CCD) is well suited to the methodology since it is comprised of the first order orthogonal design and the addition of axial points outside the first order design region of experimentation. A Rotatable Design is defined to be a design in which the variance of the estimated response is a function only of distance from the center of the design and not of the direction from the center. A Uniform Precision Design is defined to be a design in which the precision of $\hat{y}$,

$$\rho(\hat{y}) = \frac{N\mathrm{Var}(\hat{y})}{\sigma^2}, \tag{2.10}$$

at the design center is equal to the precision at a radius $\rho = 1$. Philosophically, this means that the estimated response receives uniform importance with the region $\rho = 1$. Table 2 depicts the choice of number of first order design points (F), axial points (n_a), center points (n_2), total points (N), and displacement distance of axial points (α) for Uniform Precision (up) and Orthogonal Rotatable CCD (ortho) of a varying number of unknown (k) (47).

Once a design has been selected and the data collected, least squares regression is performed to yield Equation 2.9. An ANOVA and lack-of-fit test is then conducted. If there is significant lack-of-fit, the experimenter can either fit a higher order response function or adjust his region of experimentation until the second order response function is adequate. Equation 2.9 can also be expressed in matrix notation as

$$\hat{y} = b_0 + \underline{x}'\underline{b} + \underline{x}'B\underline{x} \tag{2.11}$$

Table 2. Uniform Precision and Orthogonal Rotatable Central Composite Designs. From Myers (47).

k	2	3	4	5	5 ½ rep	6	6 ½ rep	7 ½ rep	8 ½ rep
F	4	8	16	32	16	64	32	64	128
n_a	4	6	8	10	10	12	12	14	16
n_2 (up)	5	6	7	10	6	15	9	14	20
n_2 (orth)	8	9	12	17	10	24	15	22	33
N (up)	13	20	31	52	32	91	53	92	164
N (orth)	16	23	36	59	36	100	59	100	177
α	1.414	1.682	2.000	2.378	2.000	2.828	2.378	2.828	3.364
λ_4 (up)	0.81	0.86	0.86	0.89	0.89	0.91	0.90	0.92	0.93
λ_4 (orth)	1.00	0.99	1.00	1.01	1.00	1.00	1.01	1.00	0.998

where

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_k \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} b_1 \\ b_2 \\ \cdots \\ b_k \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} & b_{12}/2 & \cdots & b_{1k}/2 \\ & b_{22} & \cdots & b_{2k}/2 \\ & & \cdots & \cdots \\ \text{sym} & & \cdots & b_{k-1,k}/2 \\ & & & b_{kk} \end{bmatrix} \tag{2.11}$$

Elementary calculus optimization of Equation 2.11 yields an estimated point of maximum response, termed the stationary point, given by

$$\underline{x}_0 = -B^{-1}\underline{b}/2 \tag{2.12}$$

The stationary point can lie inside or outside the region of experimentation. It is not advisable to extrapolate the response function outside the region of experimentation.

When analyzing a multiple response system, the extrapolation caveat assumes great importance. If the optima of all responses are in

one region of experimentation there is no cause for concern. If second order response equations cannot be fitted for all response in the same region of experimentation, two courses of action are available. First, the experimenter may choose a primary response and utilize its region of experimentation to fit first order models to those responses which are not optimum in the chosen region. Second, the experimenter may choose a compromise region of experimentation between the optima and fit first order models for the responses in this region. One must be careful not to extrapolate for any response outside its region of experimentation.

To facilitate interpretation of the second order fitted response function, the experimenter can perform a canonical analysis. Initially the response function is translated from the origin to the stationary point. Next the axes are rotated to correspond to the principle axes of response surface. To translate Equation 2.11 to origin $\underline{x}_0$, the transformation

$$\underline{z} = \underline{x} - \underline{x}_0 \tag{2.13}$$

is made resulting in

$$\hat{y} = b_0 + \underline{x}_0'\underline{b} + \underline{x}_0'B\underline{x}_0 + \underline{z}'\underline{b} + \underline{z}'B\underline{x}_0 + \underline{x}_0'B\underline{z} + \underline{z}'B\underline{z}. \tag{2.14}$$

By defining the estimated response at the stationary point as

$$\hat{y}_0 = b_0 + \underline{x}_0'\,\underline{b}/2, \tag{2.15}$$

Equation 2.14 becomes

$$\hat{y} = y_0 + \underline{z}'B\underline{z}. \tag{2.16}$$

An orthogonal transformation,

$$z = M\underline{w}, \tag{2.17}$$

is then made such that

$$\begin{aligned} \underline{z}'B\underline{z} &= \underline{w}'M'BM\underline{w} \\ &= \sum_{i=1}^{k} \lambda_i w_i^2 \end{aligned} \tag{2.18}$$

where λ_i, i 1,2,...,k, are the eigenvalues of matrix B. By substitution of Equations 2.14, 2.16, and 2.18 the canonical form of Equation 2.11 is

$$\hat{y} = y_0 + \sum_{i=1}^{k} \lambda_i w_i^2. \tag{2.19}$$

Interpretation of the response function is based on the λ_i of Equation 2.19. If all the λ_i are negative, $\underline{x}_0$ is a maximum as depicted in Figure 6(a). If all the λ_i are positive, $\underline{x}_0$ is a minimum. If the λ_i have different signs, the stationary point lies in a saddle region, as shown in Figure 6(b), and possibly indicates the existence of two maxima. If one λ_i is extremely small, the surface is a stationary ridge, as depicted in Figure 6(c), with a range of possible variable combinations yielding an approximately optimum response. Should $\underline{x}_0$ lie outside the region of experimentation, the surface approaches the shape of a rising ridge as shown in Figure 6(d). The relative magnitudes of the λ_i indicates elongation or contraction of the response surface in various directions. Figure 7 shows various response surfaces for the three inde-

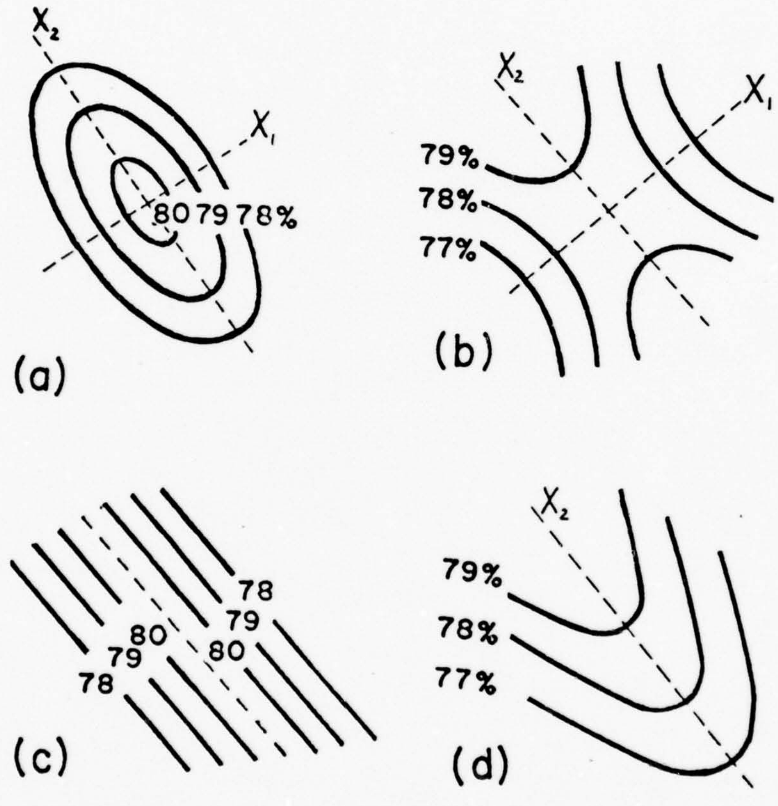

Figure 6. Response Surfaces Generated by a Second Degree Equation With Two Independent Variables. Note: x_i in this figure is equivalent to λ_i in the text. From Box[1] (10)

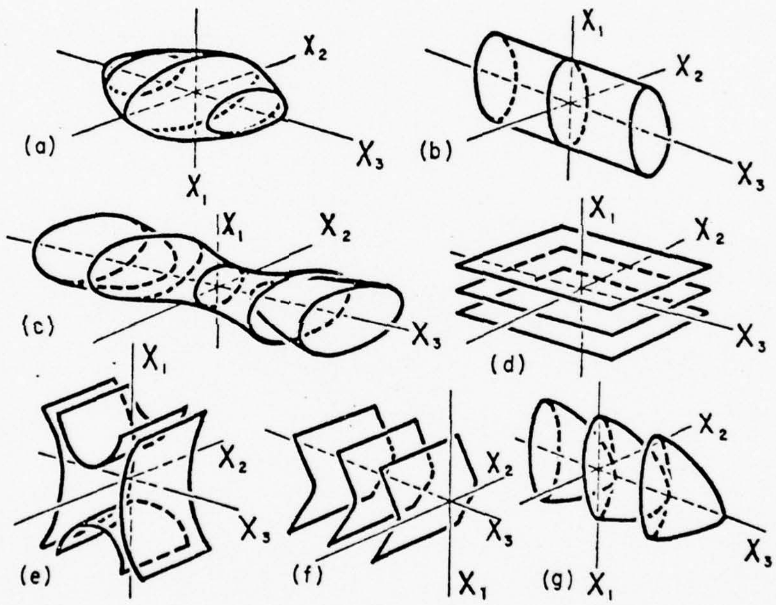

Figure 7. Response Surfaces Generated by a Second Degree Equation With Three Independent Variables. Note: x_i in this figure is equivalent to λ_i in the text. Figures above have the following λ_i: (a) --- or +++, (b) --0, (c) --+, (d) -00, (3) -0+, (f) -00 $\underline{x}_0$ at ∞, (g) --0 $\underline{x}_0$ at ∞. From Box (10).

pendent variable case.

The foregoing review of response surface methodology is intended to familiarize the reader with concepts utilized in Chapter IV of this research. Should the reader desire additional information on the subject, the text by Myers (47) is recommended as a definitive work.

Multiple Response Surface Optimization Literature Survey

In many practical applications of response surface methodology, more than one response function is generated by the independent variables. For instance, a chemical reaction with independent variables such as amount of reactants, temperature, and pressure may have multiple response functions such as purity, amount of yield, and cost. Each response function will be in the form of Equation 2.6 or 2.9. Confronted with multiple response functions, the decision maker cannot apply simple unifunction optimization. Research on multiple response surface optimization was rather sparse prior to the development of mathematical programming methodology. Each contribution to mathematical programming is ensued by its application to multiple response surface optimization. Thus far, the efforts seem to divide into two classes which could be termed multiple objective optimization and constrained single objective optimization.

Initial efforts were directed toward the graphical optimization of multiple response surfaces. Box (18), in 1954, cites an example of a chemical reaction where two reactants, A and B, formed a mixture of C and D. The objective was to maximize C while constraining D to be less than 20%. Canonical analysis indicated that C was maximized along a plane of 68% yield, as shown in Figure 8. A second response function was derived

for D and set equal to 20%. As shown in Figure 9, the constraint response function was superimposed on the maximum yield plane, allowing a visual choice of an optimum operating point. Box (10) also recognized that ridge systems, offering a wide choice of independent variable settings with minimal effect on the dependent response, are extremely useful in this type of optimization. For a three variable system, he shows a three dimensional grid which could display contours and assist in visual optimization. Line (42) refined this technique by use of acetate plates with the response surfaces drawn on them. Two articles by Hunter (35,36), in 1956, also describe graphical analysis as an optimization technique.

As mathematical programming methodology was developed, its application to response surfaces was obvious. Schrage (53), in 1957, utilized linear programming to assist in optimization of a Catalytic Cracking operation. The gradient of the objective response was maximized in the presence of the gradients of constraint responses and bounds on the independent variables. This optimum direction was then followed in the steepest ascent search. Linear programming could be utilized since the gradients of second order response functions are linear.

Quadratic response surfaces were optimized directly by Umland and Smith (57), in 1959, through the use of LaGrange Multipliers. Yield, Equation 2.20, was selected as the primary response and maximized constrained by fixed maximum values of the secondary response purity, Equation 2.21.

$$\hat{y}_p = 55.84 + 7.31x_1 + 26.65x_2 - 3.03x_1^2 - 6.96x_2^2 + 2.69x_1x_2 \quad (2.20)$$

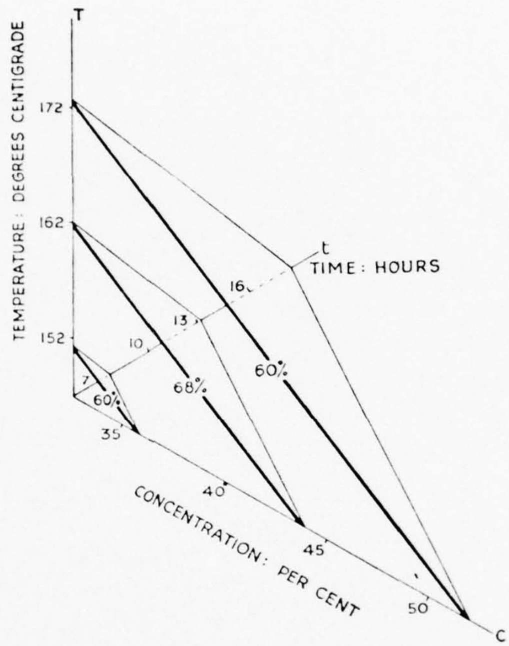

Figure 8. Yield Planes of Box Experiment
From Davies (18)

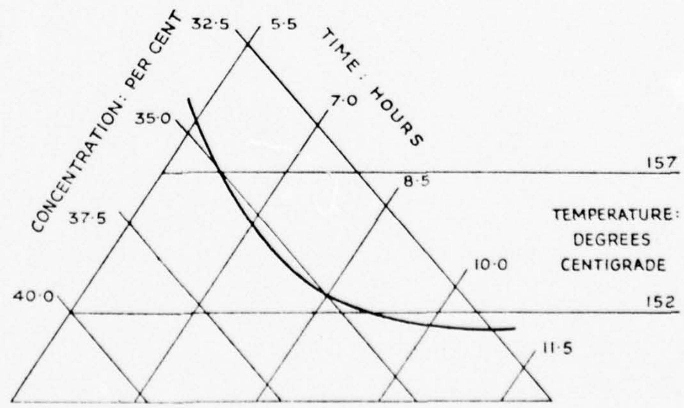

Figure 9. Superimposition of Constraint Response on Primary Response in Box Experiment. From Davies (18)

$$\hat{y}_s = 85.72 + 21.85x_1 + 8.59x_2 - 9.20x_1^2 - 5.18x_2^2 - 6.26x_1x_2 \qquad (2.21)$$

The response surfaces are graphed in Figure 10 and results are listed in

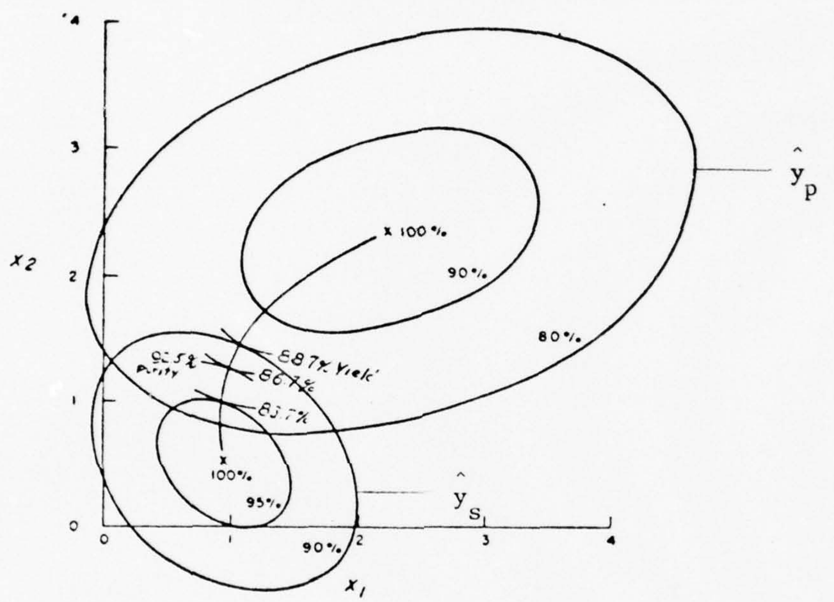

Figure 10. Umland-Smith Response Surfaces From Umland and Smith (57).

Table 3. By setting the secondary response equal to maximum values, equality constraints are created. In 1963 Michaels and Pengilly (43) also utilized LaGrange Multipliers to achieve maximum yield constrained by a fixed maximum cost function. The cost function was algebaically derived. Chow (16) demonstrated that the same technique could be utilized with inequality constraints. He also simplified the computational procedure by eliminating the need to solve a set of simultaneous equations through use of a transformation.

Hoerl (34), in 1959, introduced two techniques to the literature.

Table 3. Umland-Smith Optimization Results.

Purity Maximum	94.87 95.0	92.47 92.5	89.995 90.0
Yield	83.66	86.73	88.68
X_1	0.965	1.005	1.075
X_2	1.088	1.316	1.479

The first is an extension of graphical analysis to ridge analysis with more than two independent variables. One response is maximized or minimized while constrained by an upper bound on the second response. The variables are constrained to fall on the sphere of radius R by

$$\sum_{i=1}^{n} x_i^2 = R \tag{2.22}$$

and ridge analysis is iteratively performed, starting with the independent variable values which optimize the objective response, until the constraint responses are satisfied. The second technique is a multiple objective technique where the multiple responses are combined into one response by use of subjective weightings. Montgomery, Talavage, and Mullen (46), in 1971, pursued the weighting technique in the multiple response surface optimization of a traffic network computer simulation. Two responses, average delay per vehicle and average stop per vehicle, were linearly combined by transforming both to seconds of delay. This composite response was optimized according to the techniques discussed

in the first section of this chapter.

Nonlinear programming techniques are readily adapted for use in constrained optimization of multiple response surfaces. Carroll (14), in 1960, devised the Created Response Surface Technique which incorporates the constraint responses into the objective response by the use of a penalty function. As the steepest ascent optimization approaches the boundaries of the constraints, the objective function is penalized at a greater rate. Thus, through the sequential application of unconstrained optimization techniques, the stationary point is reached without violating the constraints. This technique was a forerunner of barrier and penalty function techniques in nonlinear programming. In 1960 Box (11) advocated the use of linear programming for the solution of multiple response chemical problems.

Lind, et al, (41) applied the graphical analysis technique to optimize the system shown in Figure 11. The two responses were cost and yield of a pharmaceutical process of American Cyanamid Company. A similar optimization of cost and yield was performed on a liquor fermentation process by Remmers and Dunn (51). Smith and Rose (55), in 1963, utilize the graphical technique with an interesting modification. One response is is a usual empirically determined equation while two other response equations are from subjective ratings. Graphical analysis was also utilized by Wu (68) in tool life testing, Ellis, et al, (22) in Raschig synthesis of Hydrazine, and Taraman and Lambert (56) in selection of machining variables. The graphical technique can and has served as both a multiple objective and a constrained optimization technique.

While analyzing the design of extruder screws, Underwood (58)

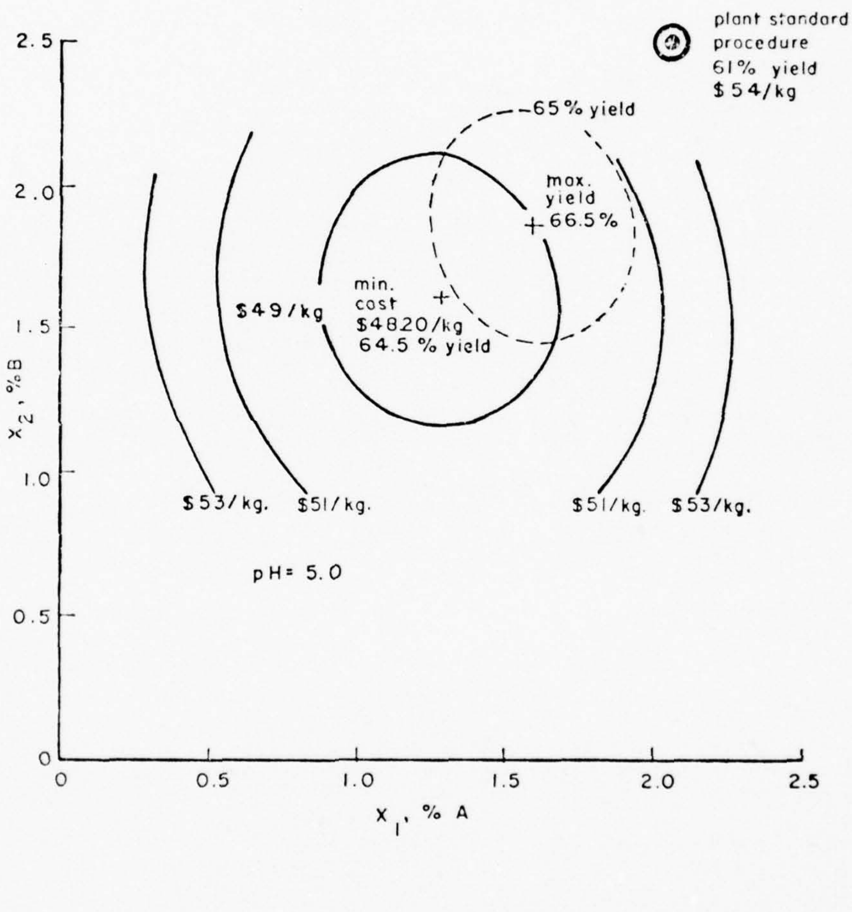

Figure 11. Lind, et al, Cost and Yield Response Contours. From Lind, et al, (41)

suggested that the advent of computers allowed for an enumerative search for the optimum of a multiple response system. Bolker (9) utilized this technique in studying delignification by Nitrogen compounds. He set one response at consecutive values and solved the response functions simultaneously.

As nonlinear programming progressed, so did its application to multiple response surface optimization. Baily, et al, (3) applied nonlinear optimization to the kraft pulping process. Responses such as yield, brightness, and Kappa number were optimized by an, unfortunately, undisclosed nonlinear technique. A method termed cheapest ascent was developed by Heller and Staats (30), in 1973. They combined a yield response and a cost constraint response into a profit objective response. Since the value of the gradient is dependent upon the metric used, a common scale of equal costs per unit change was adopted. Constraints on the system were both algebraic and response surface functions. The system was optimized utilizing Zoutendijk's method of feasible directions.

The LaGrange Multiplier approach was modified by Myers and Carter (48), in 1973. They did not equate the constraint response to a specific value, but rather devised a methodology which allowed a graphical display of optimal primary response solutions for varying values of the constraint response. Two problems were solved in the article. The first consisted of three independent variables with region constraints

$$-2.5 \leq x_i \leq 2.5 \quad (i = 1,2,3) \tag{2.23}$$

forming the dual responses

$$\hat{y}_p = 65.39 + 9.24x_1 + 6.36x_2 + 5.22x_3 - 7.32x_1^2 - 7.76x_2^2 - 13.11x_3^2 - 13.68x_1x_2 - 18.92x_1x_3 - 14.68x_2x_3 \quad (2.24)$$

$$\hat{y}_s = 56.42 + 4.65x_1 + 8.39x_2 + 2.56x_3 + 5.25x_1^2 + 5.62x_2^2 + 4.22x_3^2 + 8.74x_1x_2 + 2.32x_1x_3 + 3.78x_2x_3 \quad (2.25)$$

Figure 12 is solved for $\hat{y}_p$ given a value of $\hat{y}_s$. Values of the independent variables are then obtained from Figure 13. With $\hat{y}_s = 65.0$, $\hat{y}_p$ was maximized at $x_1 = 2.07$, $x_2 = -1.15$, and $x_3 = -0.6$, yielding a response of approximately 74.0. A second problem was solved incorporating spherical region constraints necessitated by an unbounded primary response within the constraint response region. Figure 14 shows the response surfaces of the equations

$$\hat{y}_p = 53.69 + 7.26x_1 - 10.33x_2 + 7.22x_1^2 + 6.43x_2^2 + 11.36x_1x_2 \quad (2.26)$$

$$\hat{y}_s = 82.17 - 1.01x_1 - 8.61x_2 + 1.40x_1^2 - 8.76x_2^2 - 7.20x_1x_2 \quad (2.27)$$

Two constraints are imposed,

$$84 < \hat{y}_s < 88 \quad (2.28)$$

and

$$x_1^2 + x_2^2 \leq 1. \quad (2.29)$$

The primary response was maximized at 67.0 while $\hat{y}_s = 87.8$ and $x_1 = 0.85$ and $x_2 = 0.6$. Since this method is graphical, it is limited to two response equations without undue difficulty of interpretation. Also the B

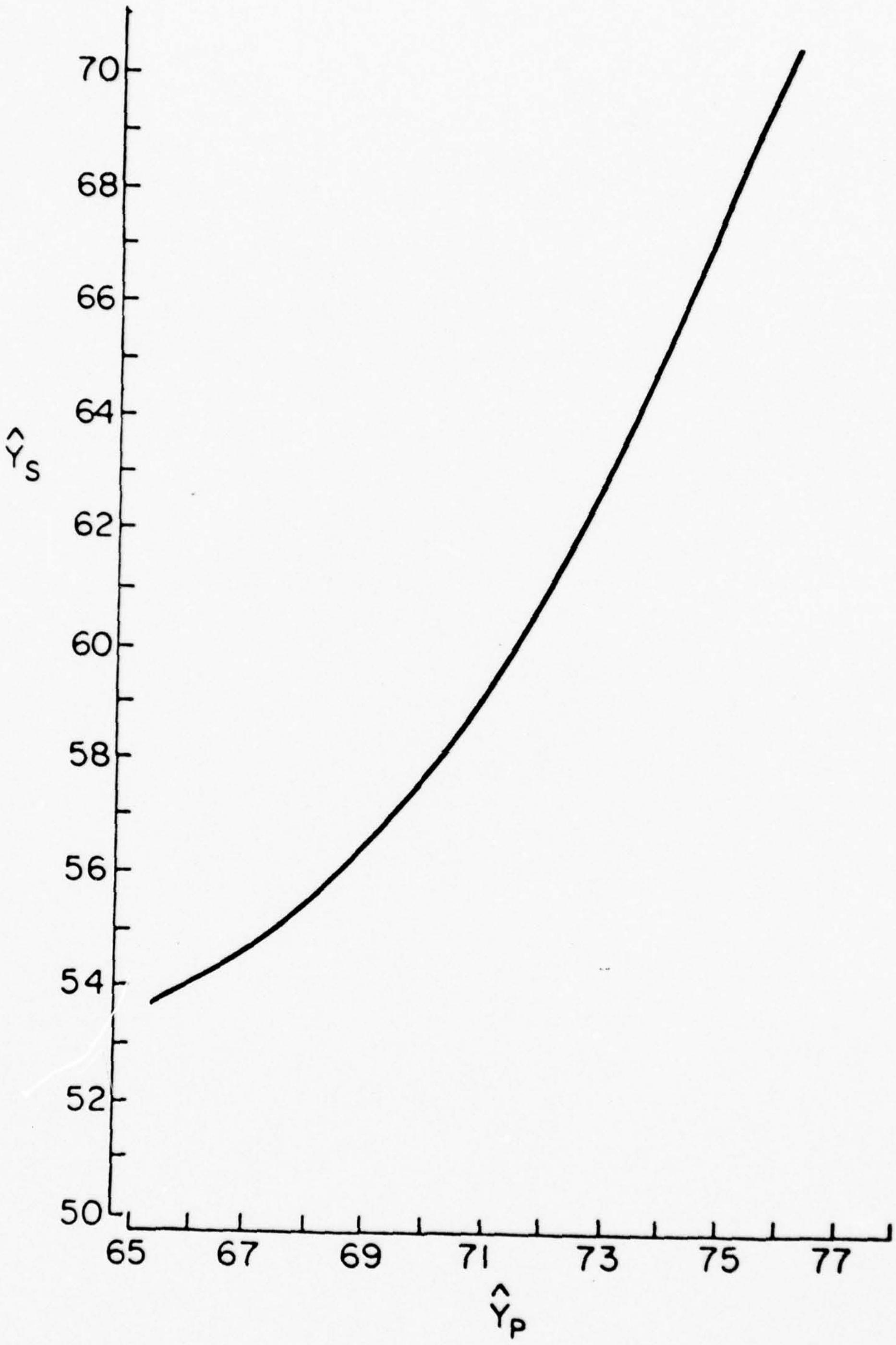

Figure 12. Maximum Estimated Primary Response at Specific Values of the Constraint Response. From Myers and Carter (48)

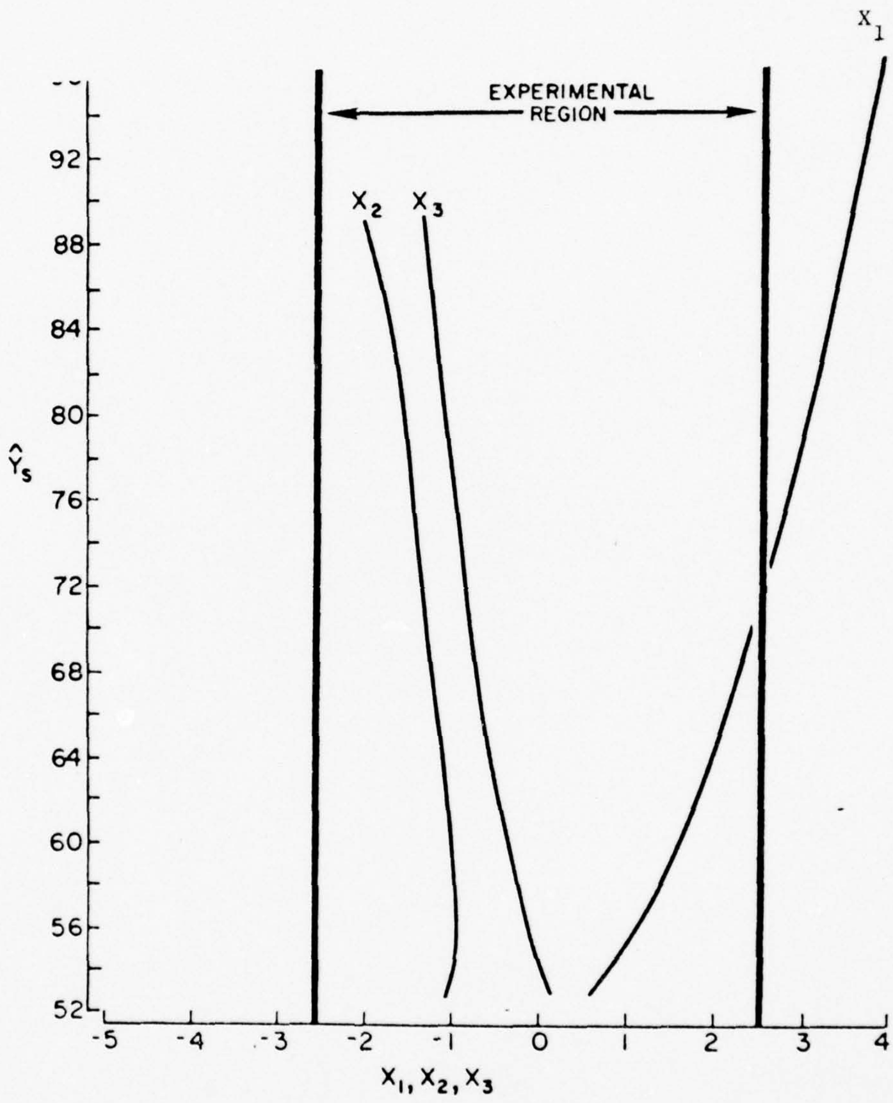

Figure 13. Conditions of Constrained Maxima on Primary Response for Fixed Values of $\hat{y}_s$. From Myers and Carter (48)

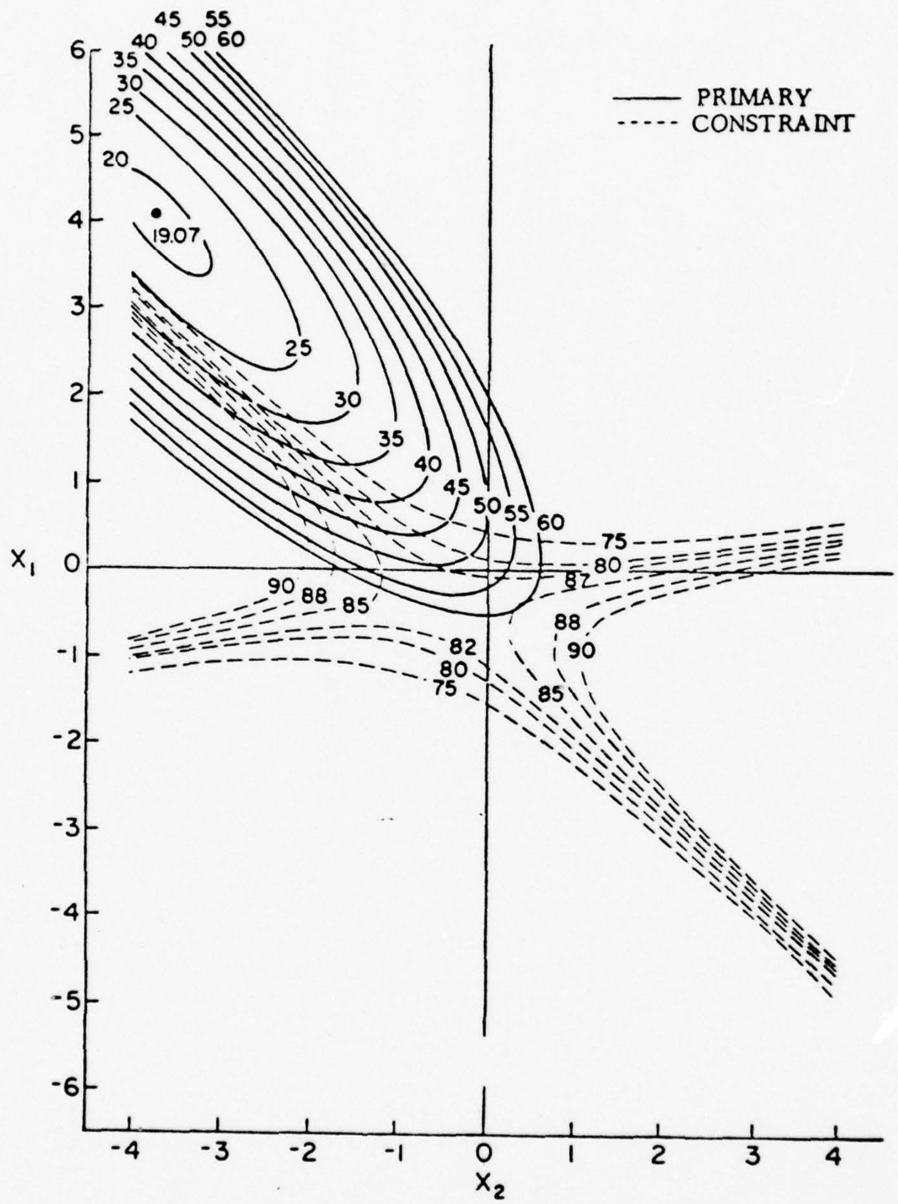

Figure 14. Response Surface of Myers and Carter Problem Two. From Myers and Carter (48)

matrix of both responses, shown in Equation 2.11, cannot be indefinite or solution is impossible by this method.

Further application of nonlinear programming was accomplished by Fields (23), in 1974. He utilized the Hooke and Jeeves Pattern Search Technique, diagramed in Figure 15, to optimize versions of the Umland and Smith and Myers and Carter problems discussed previously in this section. Fields examined three formulations of the response systems:

1. A single objective function with other response functions treated as constraints and explicitly set to a fixed value.

2. A single objective function with implicit, penalty function type consideration of the other response functions as constraints.

3. A weighting function combination of all response functions into a single function.

He concluded that the first formulation was unsatisfactory due to the inability to slightly violate the constraints. The second formulation was an improvement, though requiring numerous computer iterations from various starting points with varying penalty sizes. Fields found the most promise in the weighting scheme as an aid to the decision maker. In his research, however, various weights were applied with solutions displayed in tabular format. Once again the computer runs required are considerable and the assistance of an expert is necessary. His results are compared to the original authors' results in Table 4.

A recent addition to the literature is the work of Biles (8), in 1975, which utilizes the gradient projection technique of nonlinear programming. A primary response is optimized while secondary responses are constrained within specified bounds. The technique is mainly the usual

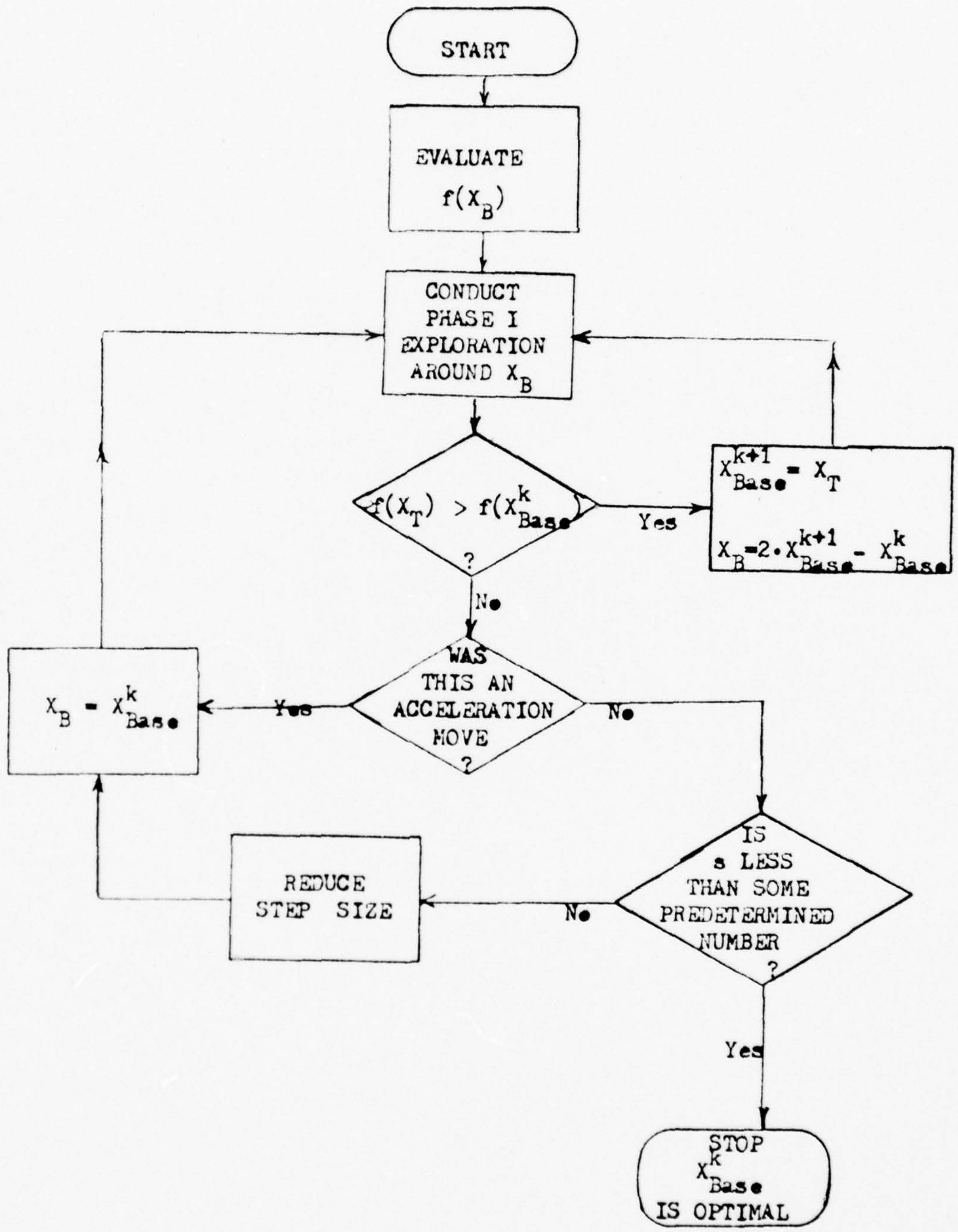

Figure 15. Hooke and Jeeves Pattern Search Technique Flow Chart. From Fields (23)

Table 4. Comparison of Fields' and Original Results

Variable	Umland and Smith	Fields	Myers and Carter
$\hat{y}_{p1}$	83.66	83.4562	
$\hat{y}_{s1}$	94.87	94.9992	
x_{11}	0.965	0.96643	
x_{21}	1.088	1.07373	
$\hat{y}_{p2}$	86.73	86.6441	
$\hat{y}_{s2}$	92.47	92.4998	
x_{12}	1.005	1.00562	
x_{22}	1.316	1.31055	
$\hat{y}_{p3}$	88.68	88.6623	
$\hat{y}_{s3}$	89.995	89.9997	
x_{13}	11.075	1.08223	
x_{23}	1.479	1.47497	
$\hat{y}_{p1}$		73.9145	73.66
$\hat{y}_{s1}$		64.9997	65.22
x_{11}		2.1250	2.07
x_{21}		-1.25	-1.15
x_{31}		-0.6222	-0.6
$\hat{y}_{p2}$		67.5716	67.8
$\hat{y}_{s2}$		87.8056	88.19
x_{12}		0.60	0.85
x_{22}		-0.80	-0.6

gradient search optimization unless the gradient direction leads out of the feasible region described by the constraint responses. Should this occur, gradient projection is used to bring the search direction back into the feasible region.

As can be seen from this literature survey, most of the research in multiple response surface optimization has been devoted to constrained optimization techniques utilizing various nonlinear programming algorithms. Such approaches require selection of a primary response with relegation of other responses to constraint status. The application of these approaches to more than three responses has not been demonstrated. The military decision maker may well desire to array the importance of multiple responses in a more controlled manner. Thus this research is devoted to the application of a multiple objective optimization technique to the multiple response problem.

Multiple Objective Optimization Literature Survey

Charnes and Cooper (15), in 1961, proposed goal programming as a solution technique for multiple linear objectives with linear constraints. If x_1, x_2, ..., x_n are a set of subgoals to be achieved and a_1, a_2 ..., a_n are technological coefficients, then the objective function is

$$f(x_1,x_2,\ldots,x_n) = a_1x_1 + a_2x_2 + \ldots + a_nx_n \qquad (2.30)$$

The constraints can be expressed in the form

$$a_ix_i = b_i \quad (i = 1,2,\ldots,n) \qquad (2.31)$$

where b_i is the i th goal value. Deviation above or below a goal is ac-

commodated by the slack variables y_i^+ or y_i^- respectively. The goal programming problem is then expressed as

$$\begin{aligned} \text{Min } Z &= \underline{y}^+ + \underline{y}^- \\ \text{S.T. } A\underline{x} + \underline{y}^- - \underline{y}^+ &= \underline{b} \\ x, d^-, d^+ &\geq 0. \end{aligned} \tag{2.32}$$

Solution by usual linear programming methods will yield values of $\underline{x}$ which come closest to meeting the goal values, $\underline{b}$. Nonlinear objectives or constraints were not considered. Ijiri (37) modified the technique of Charnes and Cooper to develop the formulation stated in Equation 2.32. Since most problems would not have completely compatible goals, Ijiri proposed a weighting and ordering scheme to allow the decision maker to set goal priorities.

In 1971, Ruefli (52) extended goal programming by adapting it to linear decomposition models. He worked with goals being set at various levels in an organization. Lee (39) has been a prolific advocate of goal programming. He recognizes that goal programming is very limited in nonlinear situations and cites no examples in his text written in 1972. Lee does detail applications of linear goal programming ranging from financial decisions to academic planning to government decision analysis. Lee and Moore (40), in 1973, apply goal programming to the linear optimization of multiple objective transportation problems. In that same year Hindelang (32) discussed the application of multiple objective linear goal programming to Quality Control optimization.

Johnsen (38), in 1968, reviews the basic results of Charnes and Cooper and Ijiri prior to researching the application of computer simula-

tion to the multiple objective problem. He proposes that simulations be performed on a multiple objective system with varying limits on the objectives. This technique would apply only to situations which could be simulated in total and would require considerable computer time.

When confronted with optimization of a refinery, Seinfeld and McBride (54), chose two formulations of the multiple objective problem. Their two objectives were to maximize total profit and to minimize the sensitivity of profit to variations in refinery conditions. The first formulation was a weighted combination of the two objectives. The second approach was to maximize the primary objective, then minimize the second objective while constraining the displacement of the solution from the primary optimum. Zoutendijk's method of feasible direction was used for the nonlinear optimization. The first formulation requires an initial subjective weighting by the decision maker. The second approach implies a primary objective and a secondary objective which will be violated by an uncontrollable amount.

Another approach to the linear multiple objective problem is POP, Progressive Orientation Procedure, devised by Benayoun, Tergny, and Keuneman (7), in 1970. This is a sequential procedure of weighted linear optimizations integrated interactively with the decision maker. By answering questions concerning the current optimum, the decision maker influences the location of the next optimization. Their algorithm, STEM, is confined to linear problems. Geoffrion (27) utilized a similar philosophy in Vector Maximal Decomposition Programming. He uses an implicit preference function to combine multiple nonlinear objectives. The perference function is determined interactively with the decision maker.

This approach will be discussed in detail later in this section.

Multicriterion linear programming problems were examined by Belenson and Kapur (6), in 1973. They developed a two person zero-sum game approach which interacted with the decision maker to determine disparities between the solution and his preferences. Monarchi, Kisiel, and Duckstein (45) developed an algorithm termed a sequential multiobjective problem solving technique, SEMOPS, to interactively solve multiple objective nonlinear goal programming problems. The algorithm involves a surogate objective function

$$\text{Min } s = \sum_{t \varepsilon T} d_t \tag{2.33}$$

where d_t reflects whether a goal has been satisfied. SEMOPS presents the decision maker with alternatives from which to choose. The approach is very similar to the algorithm adopted by this research. Vemuri (65), in 1974, developed an algorithm which sought a noninferior solution set rather than an optimum solution. It is based on deriving the Pareto optimal set, that is, the line from which a deviation will improve no objective function. Currently this algorithm is limited to specific formulations of the objective functions and no constraints.

The multiple objective optimization algorithm adopted by this research is the Geoffrion-Dyer Interactive Vector Maximal algorithm. Chapter III of this research will detail the algorithm, thus the following will be a description of its development. The early theoretical work by Geoffrion (27) has previously been discussed. Geoffrion and Hogan (29), in 1972, formalized an algorithm and applied it to two-level organizations with multiple objectives. An overall objective function of the decision

maker's utility function is optimized without explicit knowledge of the function. Marginal rate of substitution indifference tradeoffs between objectives, interactively developed by the decision maker, are transformed into point gradients of his utility function. These are maximized, subject to region definition constraints, to product an optimal direction vector. The decision maker then selects an optimal solution along this vector. Linearity is not a requirement in objectives or constraints.

Dyer (21) adapted the algorithm to Interactive Goal Programming. Nonlinear functions were applicable to the algorithm but Dyer cautioned that his adaptation, "... can be expected to provide an optimal solution to the multiple criteria problem only in restrictive special cases." He found value in the insights and alternatives which the algorithm presented to the decision maker. Garrido (26), in 1974, altered the suboptimization portion of this algorithm by utilizing LaGrange Multipliers in an application to Multi-Item Inventory systems.

In December 1972, Geoffrion, Dyer, and Feinberg (28) formalized the basic algorithm. An article was published detailing the algorithm and its application to the operation of an academic department. Dyer (19), in 1973, published an article describing an ALGOL computer program of the algorithm. He displayed output, Figure 16, of the algorithm optimizing an automobile purchase decision. In a later article (20), he describes an experiment with graduate student subjects knowledgeable in mathematical programming, solving the automobile problem with various algorithms. The Vector Maximal algorithm received unanimous subjective praise for ease of use and comprehension. Most recently, Courtney (17) has drafted a paper

```
                 A         B
Cost          2150      2100
HP             108.3     108.3
MPG             29        27
Which do you prefer?  If you are indifferent, Type I
A
                 A         B
Cost          2150      2100
HP             108.3     108.3
MPG             29        28
Which do you prefer?  If you are indifferent, Type I
B
                 A         B
Cost          2150      2100
HP             108.3     108.3
MPG             29        27.5
Which do you prefer?  If you are indifferent, Type I
I
```

Estimation of w_3^k (Decision Maker's Tradeoff between f_1 and f_3)

```
The Tradeoffs are
Cost             -1
HP               10
MPG              33.33
```

The vector w^k

```
New Operating Point
              2500       140        20.5
New Decision Vector
                 1.5      10       200
```

Computed by Frank-Wolfe Algorithm (5)

```
Enter Number of Points to See in Step Size-Problem
⎕:
                 7

Select a Preferred  F  Vector from the Following Rows
              2150      108.3      29
              2208      113.6      27.58
              2267      118.9      26.17
              2325      124.2      24.75
              2383      129.4      23.33
              2441      134.7      21.91
              2500      140        20.5
```

"Step-Size" determination

```
Enter New Point
⎕:
              2325      124.2      24.75
```

f^{k+1}

```
If you wish to end iterations, Type 'E.'  Otherwise, Type 'C.'
C

Enter Desired Perturbations
⎕:
              -100       10         2
```

Δf^{k+1}

Figure 16. Sample Output From the VM (Vector Maximal) Program. From Dyer (19)

applying the algorithm to capital appreciation and income portfolio selection.

This brief survey of multiple objective optimization has revealed a majority of effort on the linear problem. The work of Geoffrion and Dyer stands out in the nonlinear problem area. Utilization of the Interactive Vector Maximal algorithm would allow participation of the military decision maker in the optimization process. His military experience and expertise would be utilized in making controlled marginal rate of substitution decisions. After an optimal direction is determined, the military decision maker would perform the uni-directional search optimization. In this alliance between military decision maker and mathematical programming, the "black box" fixed solution syndrome is alleviated if not eliminated. Since all alternatives are presented to the decision maker in the dependent response space rather than the independent variable space, a multitude of alternate solutions are considered. An application of the Geoffrion-Dyer Interactive Vector Maximal algorithm to multiple response surface optimization would seem to generate favorable dividends. It is in that direction which this research will now proceed.

CHAPTER III

DEVELOPMENT OF AN OPTIMIZATION METHODOLOGY

The Frank-Wolfe Linear Approximation Algorithm

The theoretical basis of the Geoffrion-Dyer Interactive Vector Maximal algorithm is the Frank-Wolfe Linear Approximation algorithm. Development of a methodology involving the latter algorithm must therefore begin with the former. The Frank-Wolfe algorithm (69) solves the nonlinear programming problem

$$\begin{aligned} &\text{Max } f(\underline{x}) \\ &\text{S. T. } A\underline{x} \leq \underline{b} \\ &\underline{x} \geq 0 \end{aligned} \tag{3.1}$$

by means of linear approximations. The linear approximation to $f(\underline{y})$, where $\underline{y}$ is a solution to Equation 3.1, at the feasible point $\underline{x}^k$ is $f_L(\underline{y})$ where

$$f_L(\underline{y}) = f(x^k) + \nabla f(\underline{x}^k)^t(\underline{y}-\underline{x}^k). \tag{3.2}$$

The algorithm seeks to maximize the linear approximation of the objective function within the constraint set. By substitution of Equation 3.2, Equation 3.1 becomes

$$\begin{aligned} &\text{Max } f(\underline{x}^k) + \nabla f(\underline{x}^k)^t(\underline{y}-\underline{x}^k) \\ &\text{S. T. } A\underline{y} \leq \underline{b} \\ &\underline{y} \geq 0 \; . \end{aligned} \tag{3.3}$$

Futher simplification is possible by realizing that $\underline{x}^k$ is a fixed feasible point throughout an iteration of the algorithm, rendering several terms in the objective function constant.

The final form of Equation 3.3 is

$$\begin{aligned} &\text{Max } \nabla f(\underline{x}^k)^t \cdot \underline{y} \\ &\text{S.T. } A\underline{y} \leq \underline{b} \\ &\qquad \underline{y} \geq 0 \quad . \end{aligned} \tag{3.4}$$

The optimum $\underline{y}^k$ of Equation 3.4 is constrained to be feasible and is the maximum of the linear approximation of the original objective function. An improved value of f should lie on a direction $\underline{d}^k$ from $\underline{x}^k$ to $\underline{y}^k$:

$$\underline{d}^k = \underline{y}^k - \underline{x}^k . \tag{3.5}$$

A uni-direction search is therefore conducted along

$$\underline{x}^k + \tau(\underline{y}^k - \underline{x}^k) \quad 0 \leq \tau \leq 1 \tag{3.6}$$

to yield an improved and feasible $\underline{x}^{k+1}$ for the next iteration of the algorithm. The algorithm terminates at solution point $\underline{x}^*$ if $\underline{y}^*$, the solution to Equation 3.4 where $\underline{x}^k = \underline{x}^*$, implies

$$\nabla f(\underline{x}^*)^t(\underline{y}^* - \underline{x}^*) \leq 0 . \tag{3.7}$$

By substituting Equation 3.5 into 3.7 it is seen that $\underline{x}^*$ satisfies the Kuhn-Tucker conditions that are necessary for optimality. Farkas'Lemma (69) states that

$$q^t\underline{x} \leq 0 \tag{3.8}$$

for all x such that $A\underline{x} \leq 0$ is equivalent to the statement that there exists $\underline{u} \geq 0$ such that

$$q + A^t\underline{u} = 0. \tag{3.9}$$

In Equation 3.4,

$$A\underline{y} \leq \underline{b} \rightarrow A\underline{y}-\underline{b} \leq 0, \tag{3.10}$$

thus

$$\nabla(A\underline{y}-\underline{b}) \leq 0. \tag{3.11}$$

Substituting Equation 3.7 and 3.11 into 3.8 yields this version of 3.9:

$$\nabla f(\underline{x}^*) + \nabla\ (A\underline{y}-\underline{b})\underline{u} = \theta. \tag{3.12}$$

which are the Kuhn-Tucker conditions necessary for optimality.

Zangwill (69) proves the following Convergence Theorem for a non-linear programming problem:

Let the point-to-set map $A:V \rightarrow V$ determine an algorithm that given at point $Z' \varepsilon V$ generates the sequence $\{\underline{z}^k\}_1^\infty$. Also let a solution set $\Omega \subset V$ be given.

Suppose
(1) All points $\underline{z}^k$ are in a compact set $X \subset V$.
(2) There is a continuous function $Z:V \rightarrow E^1$ such that:
(a) if $\underline{z}$ is not a solution, then for an $y \varepsilon A(\underline{z})$

$$Z(\underline{y}) > Z(\underline{z})$$

(b) if $\underline{z}$ is a solution, then either the algorithm terminates or for any $\underline{y} \varepsilon A(\underline{z})$

$$Z(\underline{y}) \geq Z(\underline{z})$$

and

(3) The map A is closed at $\underline{z}$ if $\underline{z}$ is not a solution.

Then either the algorithm stops at a solution, or the limit of any convergent subsequence is a solution.

The foregoing Frank-Wolfe algorithm will now be shown to be convergent (69). An assumption must be made that f is continuous and differentiable and that the feasible region is compact. Compactness is equivalent to assuming that the feasible region is closed and bounded. By this second assumption part (1) of the Theorem is proved since $\underline{x}^k$ is feasible, $\underline{y}^k$ is feasible, and any point on a straight line between them is feasible.

To prove part (2a), assume that $\underline{x}'$ is not a solution. Then let $\underline{y}'$ be the solution to Equation 3.4 with $\underline{x}^k = \underline{x}'$. Since $\underline{x}'$ is not a solution Equation 3.7 becomes

$$\nabla f(\underline{x}')^t(\underline{y}'-\underline{x}') > 0. \tag{3.13}$$

But Equation 3.13 states that $\underline{d}'$ is an improving direction for f. Let $\underline{w}$ be a point on $\underline{d}'$ within Equation 3.6. Then

$$f(\underline{w}) > f(\underline{x}') \tag{3.14}$$

Part (2b) clearly holds if $\underline{z} = \underline{x}$ and $Z(\underline{z}) = f(\underline{x})$.

The final step in establishing convergence of the algorithm is proof of part (3) of the Theorem. Let

$$\underline{x}^k \to \underline{x}^\infty \tag{3.15}$$

and

$$d^k \to d^\infty. \tag{3.16}$$

Substitution of Equations 3.15 and 3.16 into 3.5 yields

$$\underline{y}^k \to \underline{y}^\infty = \underline{d}^\infty - \underline{x}^\infty \tag{3.17}$$

The algorithmic map is separated into two maps, D which determines the improving direction, and M which calculates $\underline{x}^{k+1}$ given the improving direction:

$$A = MD \tag{3.18}$$

The map M was shown to be closed in the proof of part (1). To prove D is closed, it is sufficient to show that $\underline{y}^\infty$ solves Equation 3.4 where $\underline{x} = \underline{x}^\infty$. Then since $\underline{d}^\infty = \underline{y}^\infty - \underline{x}^\infty$,

$$(\underline{x}^\infty, \underline{d}^\infty) \in D(\underline{x}^\infty).$$

Since $\underline{y}^\infty$ is one $\underline{y}^k$, it is feasible. By definition of $\underline{y}^k$

$$\nabla f(\underline{x}^k)^t(\underline{y}^k - \underline{x}^k) \geq \nabla f(\underline{x}^k)^t(\underline{y} - \underline{x}^k) \tag{3.19}$$

for any feasible $\underline{y}$. Taking the limit of Equation 3.19 as $k \to \infty$ yields

$$\nabla f(\underline{x}^\infty)^t(\underline{y}^\infty - \underline{x}^\infty) \geq \nabla f(\underline{x}^\infty)^t(\underline{y} - \underline{x}^\infty), \tag{3.20}$$

which states that $\underline{y}^\infty$ solves Equation 3.4 for $\underline{x} = \underline{x}^\infty$ since Equation 3.20 is true for all feasible $\underline{y}$. The map D is thus closed. Zangwill (69) has proven a theorem which states that if maps M and D are closed in Equation 3.18, map A is closed. This completes the proof of convergence. Wolfe (67) has done further work to establish upper and lower bounds on the rate of convergence of the Frank-Wolfe algorithm.

The Geoffrion-Dyer Interactive Vector Maximal Algorithm

The development and theoretical basis of the Interactive Vector Maximal algorithm have now been discussed. The following will be a detailed description of the algorithm (1,19,20,28,29). The multiple objective optimization problem can be stated as

$$\text{Max } U \ [f_1(x), f_2(x), \ldots, f_r(x)] \qquad (3.21)$$
$$\text{S. T. } \underline{x} \varepsilon X$$

where f_i, $i = 1,2,\ldots,r$, are distinct objective functions, X is the feasible decision variable space, and U is the decision maker's utility function defined on the range of f. The utility function U and each f_i is assumed to be concave and continuously differentiable, and U is increasing in each f_i. If some f_i are convex, that is, utility decreases for an increase in f_i, then a change of sign for that f_i will be required. The space X is assumed to be convex and compact.

Equation 3.21 can be solved by the Frank-Wolfe algorithm as follows:

Step 0. Choose an initial feasible solution $\underline{x}^k \varepsilon X$. Let $k = 1$.

Step 1. Determine an optimal solution $\underline{y}^k$ of the direction finding problem.

$$\text{Max } \nabla_{\underline{x}} U \ [f_1(\underline{x}^k), f_2(\underline{x}^k), \ldots, f_r(\underline{x}^k)] \cdot \underline{y} \qquad (3.22)$$
$$\text{S. T. } \underline{y} \varepsilon X$$

Let $\underline{d}^k = \underline{y}^k - \underline{x}^k$.

Step 2. Determine an optimal solution t^k of the step-size problem

$$\text{Max } U\ [f_1(\underline{x}^k + t\underline{d}^k),\ f_2(\underline{x}^k + t\underline{d}^k), \ldots, f_r(\underline{x}^k + t\underline{d}^k)\ ] \tag{3.23}$$
$$0 \leq t \leq 1\ .$$

if the solution is optimal, terminate. Otherwise let

$$\begin{aligned} \underline{x}^{k+1} &= \underline{x}^k + t\underline{d}^k, \\ k &= k + 1, \end{aligned} \tag{3.24}$$

and return to Step 1.

The Frank-Wolfe algorithm was chosen for its computational simplicity, its well established convergence discussed earlier, and its rapid initial rate of convergence as discussed by Amor (2,19).

An immediate difficulty in this procedure is the necessity of quantifying the gradient of the decision maker's utility function in Equation 3.22, By application of the chain rule,

$$\nabla_{\underline{x}} U \left[f_1(\underline{x}^k), f_2(\underline{x}^k), \ldots, f_r(\underline{x}^k) \right] = \sum_{i=1}^{r} \left(\frac{\partial U}{\partial f_i} \right)^k \nabla_{\underline{x}} f_i(\underline{x}^k) \tag{3.25}$$

where $\left(\frac{\partial U}{\partial f_i} \right)^k$ is the i th partial derivative of U evaluated at the point $\left[f_1(\underline{x}^k), f_2(\underline{x}^k), \ldots, f_r(\underline{x}^k) \right]$, and $\nabla_{\underline{x}} f_i(\underline{x}^k)$ is the gradient of f_i evaluated at $\underline{x}^k$. By substitution of Equation 3.25, 3.22 becomes

$$\text{Max} \sum_{i=1}^{r} \left(\frac{\partial U}{\partial f_i} \right)^k \nabla_{\underline{x}} f_i\ (\underline{x}^k) \cdot \underline{y} \tag{3.26}$$
$$\text{S. T. } \underline{y} \in X\ .$$

Except for the partial derivatives of U, the quantities in Equation 3.26

are known. The solution of Equation 3.26 is not affected by multiplication of the objective function by a scalar. Thus the objective function can be multiplied by the positive reciprocal of a $\left(\frac{\partial U}{\partial f_i}\right)^k$. As a standard convention, $\left(\frac{\partial U}{\partial f_1}\right)^k$ is utilized. The original vector

$$\left(\left(\frac{\partial U}{\partial f_1}\right)^k, \left(\frac{\partial U}{\partial f_2}\right)^k, \ldots, \left(\frac{\partial U}{\partial f_r}\right)^k\right) \tag{3.27}$$

is colinear with the new vector

$$\left(1, \left(\frac{\partial U/\partial f_2}{\partial U/\partial f_1}\right)^k, \ldots, \left(\frac{\partial U/\partial f_r}{\partial U/\partial f_1}\right)^k\right). \tag{3.28}$$

The components of Equation 3.28 are termed the marginal rates of substitution between f_1 and f_i, $i = 2,3,\ldots,r$, that is the preferred tradeoffs between objective 1 and objective i. There are several methods available to obtain the tradeoffs. The method utilized in this research is the ordinal comparison method, that is, "I prefer A to B." This method has been shown to be superior to the other methods (20). Initial perturbations of Δf_i^k, $i = 1,2,\ldots,r$, are obtained from the decision maker. These perturbations are obtained in a direction favorable to the decision maker, thus satisfying the need of sign determination for f_i discussed earlier in the initial assumptions. The first perturbation, Δf_1^k is the reference perturbation.

The decision maker is presented with two vectors, A being $f_i(\underline{x}^k)$, $i = 1,2,\ldots,r$, and B being $(f_i^k + \Delta f_1^k, f_2^k, \ldots, f_{i-1}^k, f_i^k - \Delta f_i^k, f_{i+1}^k, \ldots, f_r^k)$. If the decision maker prefers B, Figure 17(a), Δf_i^k is doubled. This is repeated until A is preferred. If A is preferred, Δf_i^k is halved. This

is repeated until B is preferred. After possibly several iterations of this procedure, the decision maker is indifferent to the ordinal comparison presented and Δf_i^{k*} is determined, Figure 17(b). This procedure is repeated until all Δf_i^{k*}, $i = 2,3,\ldots,r$, are determined.

One alternate method of determining the tradefoof is to simply ask the decision maker what change in the first objective value would exactly compensate a given change in each of the other objective values. Another method would be to place the objective function values 1 and i on axes of a graph and designate the current solution point. A reference point is then chosen and the decision maker trades off movement on one axis against the other. Probably the least desirable method would be to obtain a range of tradeoff values from the decision maker, and solve the direction finding problem with all values given. The decision maker would then choose a solution from the several generated step-size problems. It has been shown that the algorithm will converge even though errors are made in the determination of the tradeoffs as long as the errors decrease with each iteration (28). This is not unreasonable to assume since each iteration will educate the decision maker in the implications of his tradeoffs.

After the tradeoffs are determined, the approximation is made

$$w_i^k = \frac{\partial U/\partial\ f_i^k}{\partial U/\partial\ f_1^k} \simeq \frac{\Delta f_1^k}{\Delta f_i^k}\ ,\ i = 1,2,\ldots,r\ . \tag{3.29}$$

By substitution of Equation 3.29, 3.36 becomes

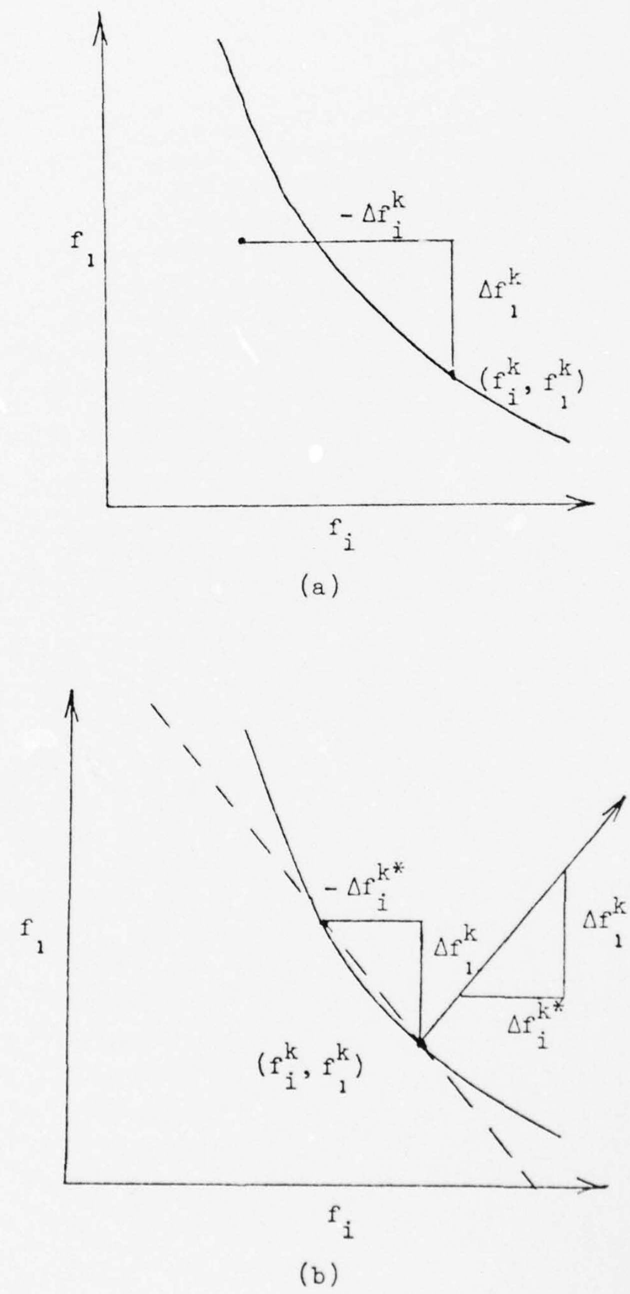

Figure 17. Estimation of w_i^k.
From Dyer (20)

$$\begin{aligned} &\text{Max } \underline{w}^{k^t} \cdot \nabla_{\underline{x}} f_i(\underline{x}^k) \cdot \underline{y} \\ &\text{S. T. } \underline{y} \in X \end{aligned} \qquad (3.30)$$

of which all quantities except $\underline{y}$ are known. Equation 3.22, and therefore Step 1 of the Frank-Wolfe method, can now be solved. Step 2 is solved by presenting the decision maker with alternatives

$$f(\underline{x}^k + t\underline{d}^k) \quad 0 \leq t \leq 1 \quad . \qquad (3.31)$$

By choosing his preferred alternative from Equation 3.31, the decision maker solves 3.23. He is then allowed to return to Step 1 or terminate the algorithm.

The Interactive Vector Maximal Algorithm is now seen to be:

Step 0. The decision maker chooses an initial point $\underline{x}^1 \epsilon X$. Let $k = 1$.

Step 1(a). The decision maker assesses his tradeoff weights w_i^k.

(b). Compute the optimal solution $\underline{y}^k$ of Equation 3.30. Let $\underline{d}^k = \underline{y}^k - \underline{x}^k$.

Step 2. The decision maker chooses an optimal t^k to Equation 3.31. If the decision maker is satisfied, terminate. Otherwise proceed as in Equation 3.24.

It is important to realize that the decision maker views the entire problem in objective value space rather than in the more confusing decision variable space. He is making tradeoffs of objectives with no distractions from the decision variables. He is also seeing a multitude of alternate solutions as he progresses through the procedure. This is an educational process for the decision maker in the implications of his tradeoffs a-

mong objectives. There is no requirement for the decision maker to be familiar with mathematical programming. It was shown earlier that the algorithm converges to an optimal solution. The decision maker may subjectively terminate the algorithm once he feels further iterations would yield minimal improvement.

Adaptation of the Interactive Vector Maximal Algorithm to the Optimization of Multiple Response Surfaces

Adaptation of the Interactive Vector Maximal Algorithm to the optimization of multiple response surfaces must begin with an examination of the algorithm's assumptions. Utility theory shows that the majority of utility functions are concave and continuously differentiable. Most multiple response problems constrain the independent design variables in one of three ways. First the variables may be given range constraints such as

$$a_i \leq x_i \leq b_i \quad i = 1,2,\ldots,k \ . \qquad (3.32)$$

These constraints are of course straight line segments and describe a convex, compact set. A second alternative would be

$$x_i + x_j \leq b_{ij} \quad i,j = 1,2,\ldots,k \ . \qquad (3.33)$$

These are also straight line segments and satisfy the assumption. The third constraint definition would be

$$\sum_{i=1}^{r} x_i^2 = b \ . \qquad (3.34)$$

These constraints describe a sphere which is convex and compact.

The assumption which is violated concerns the concavity or convexity of f_i, the response functions. As discussed in Chapter II, and pictured in Figures 6 and 7, a second order response function can take various shapes. For ease of interpretation, the two variable case will be discussed though the discussion applies to surfaces of more than two variables. If the response surface is a pure maximum or minimum, the assumptions are satisfied. If the surface is a saddle system, local and/or alternate optima might exist. In this case the algorithm is performed by choosing alternate starting points, $\underline{x}^1$, and proceeding to an optimum point in each case. A thorough procedure would be to start from each vertex of the constraint space and from the origin. Experience with the surface may dictate fewer starting points. The surface optimum would be the optimum of the local optima.

The existence of a ridge system also requires alteration of the algorithm. As long as the decision maker's usual tradeoffs lead to improvement, the algorithm proceeds normally. If the current $\underline{x}^k$ lies on the down slope of the ridge, normal tradeoffs will lead to unsatisfactory alternatives in the step-size problem. At this point the decision maker should reverse the sign of his Δf_1^k perturbation and the algorithm will bring him back up the ridge to an improved point. If the current $\underline{x}^k$ lies on the crest of the ridge, neither sign of usual perturbations will lead to improvement. At this time the decision maker must judiciously adjust the sign and magnitude of his perturbations until a different search direction is generated. This is not difficult if the interactive program displays the coefficients of Equation 3.30. The program developed for this research displays these coefficients and offers another method of

solving this problem. In the nonlinear constraint version of the program, the decision maker obtains these coefficients from the main program, terminates the main program and optimizes the suboptimization problem with another program, then returns to the main program. Upon returning to the main program, he could input a new search direction to move the current $\underline{x}^k$ off the crest of the ridge. The presence of a semi-trained analyst might be required but the procedure is not difficult. Once an $\underline{x}^k$ not on the crest is reached, usual perturbation may again be utilized. Application of the algorithm to representative problems has shown the occurance of a current $\underline{x}^k$ on the crest of a ridge to be extremely rare.

The three design variable constraint definitions, Equation 3.32, 3.33, and 3.34, yield three formulations of Equation 3.30. The w_i^k and $\nabla_{\underline{x}} f_i(\underline{x}^k)$ are known and are constants. Thus Equation 3.30 and 3.32 reduce to

$$\begin{aligned} &\text{Max} \sum_{i=1}^{r} c_i y_i \\ &\text{S. T. } a_i \le y_i \le b_i \quad i = 1,2,\ldots,r \end{aligned} \tag{3.35}$$

where $\underline{c} = \underline{w}^{k^t} \cdot \nabla_{\underline{x}} f_i(\underline{x}^k)$. Equation 3.35 can be solved by direct substitution. If c_i is positive, then set y_i at its upper bound, b_i. If c_i is negative, set y_i at its lower bound, a_i. Constraints of the type Equation 3.33 yield

$$\begin{aligned} &\text{Max} \sum_{i=1}^{r} c_i y_i \\ &\text{S. T. } A\underline{x} \le \underline{b} \ . \end{aligned} \tag{3.36}$$

This is the classic linear programming problem and can be solved by the simplex method. Constraints such as Equation 3.34 yield

$$\begin{aligned} &\text{Max} \sum_{i=1}^{r} c_i y_i \\ &\text{S. T.} \sum_{i=1}^{r} x_i^2 = b \,. \end{aligned} \tag{3.37}$$

This research used the Bazaraa Cyclic Coordinate Algorithm for Optimizing Penalty Functions computer program (5) to solve Equation 3.37.

The interactive optimization algorithm was programmed in FORTRAN for use on a CDC computer through two programs, listed in Appendix D. The first program is utilized for data input. As can be seen from an example run in Figure 18, the decision maker responds to interactive questions. The only analysis required is to compute gradients of the response functions. The upper and lower bounds of x_i are defining the region of experimentation utilized for the second order model and thus must coincide for all response functions. An example of the interactive optimization program is shown in Figure 19. The coefficients mentioned as aids in ridge problems are seen between the tradeoffs and the new decision vector.

Application of the Methodology to Multiple Response Surface Problems

This section will examine the application of the adapted Interactive Vector Maximal algorithm to the multiple response surface problems utilized by Fields. These problems were previously solved by Myers and Carter and Umland and Smith as discussed in Chapter II. It must be re-

```
INPUT NUMBER OF RESPONSE EQUATIONS
? 2
INPUT NUMBER OF INDEPENDENT VARIABLES (X"S)
? 3
INPUT INITIAL VALUE OF INDEPENDENT VARIABLES WITH . AND ,
? 2.,-2.,-1.
INPUT COEFFICIENTS OF RESPONSE EQUATION 1
? -7.23,-7.76,-13.11,0.,0.,-13.68,-18.92,0.,0.,-14.68,0.,
? 0.,0.,0.,0.,9.24,6.36,5.22,0.,0.,65.39
INPUT COEFFICIENTS OF RESPONSE EQUATION 2
? 5.25,5.62,4.22,0.,0.,8.74,2.23,0.,0.,3.78,0.,0.,0.,0.,
? 0.,4.65,8.39,2.56,0.,0.,56.42
INPUT RESPONSE EQUATION NAMES IN GROUPS OF TEN LETTERS
AND SPACES, RIGHT JUSTIFIED, ONE PER LINE
?        MAX
?        MIN
INPUT COEFFICIENTS OF GRADIENT F 1X 1
? -14.46,-13.68,-18.92,0.,0.,9.24
INPUT COEFFICIENTS OF GRADIENT F 1X 2
? -13.68,-15.52,-14.68,0.,0.,6.36
INPUT COEFFICIENTS OF GRADIENT F 1X 3
? -18.92,-14.68,-26.22,0.,0.,5.22
INPUT COEFFICIENTS OF GRADIENT F 1X 4
? 0.,0.,0.,0.,0.,0.
INPUT COEFFICIENTS OF GRADIENT F 1X 5
? 0.,0.,0.,0.,0.,0.
INPUT COEFFICIENTS OF GRADIENT F 2X 1
? 10.5,8.74,2.32,0.,0.,4.65
INPUT COEFFICIENTS OF GRADIENT F 2X 2
? 8.74,11.24,3.78,0.,0.,8.39
INPUT COEFFICIENTS OF GRADIENT F 2X 3
? 2.32,3.78,8.44,0.,0.,2.56
INPUT COEFFICIENTS OF GRADIENT F 2X 4
? 0.,0.,0.,0.,0.,0.
INPUT COEFFICIENTS OF GRADIENT F 2X 5
? 0.,0.,0.,0.,0.,0.
INPUT REGION OF INTEREST BOUNDARY DEFINITION, I FOR
INTEGER, L FOR LINEAR, OR N FOR NONLINEAR
? I
INPUT LOWER AND UPPER BOUNDS OF X1
? -2.5,2.5
INPUT LOWER AND UPPER BOUNDS OF X2
? -2.5,2.5
INPUT LOWER AND UPPER BOUNDS OF X3
? -2.5,2.5
     .214 CP SECONDS EXECUTION TIME
```

Figure 18. Example of Data Input Computer Program.

```
INPUT PERTURBATION OF F( 1), IN FAVORABLE DIRECTION
? .1
INPUT PERTURBATION OF F( 2), IN FAVORABLE DIRECTION
? .5
                                    A               B
P(VICTORY)                       .64600          .74600
E(BTL RDS)                      1.59400         1.59400
  TRNG HRS                     55.92260        55.92260
  TRNG RDS                     76.25000        76.25000
 TRNG COST                   6373.00000      6373.00000
WHICH DO YOU PREFER.  IF YOU ARE INDIFFERENT TYPE I.
? I
INPUT PERTURBATION OF F( 3), IN FAVORABLE DIRECTION
? -5.
                                    A               B
P(VICTORY)                       .64600          .74600
E(BTL RDS)                      1.59400         1.59400
  TRNG HRS                     55.92260        60.92260
  TRNG RDS                     76.25000        76.25000
 TRNG COST                   6373.00000      6373.00000
WHICH DO YOU PREFER.  IF YOU ARE INDIFFERENT TYPE I.
? I
INPUT PERTURBATION OF F( 4), IN FAVORABLE DIRECTION
? -5.
                                    A               B
P(VICTORY)                       .64600          .74600
E(BTL RDS)                      1.59400         1.59400
  TRNG HRS                     55.92260        55.92260
  TRNG RDS                     76.25000        81.25000
 TRNG COST                   6373.00000      6373.00000
WHICH DO YOU PREFER.  IF YOU ARE INDIFFERENT TYPE I.
? I
INPUT PERTURBATION OF F( 5), IN FAVORABLE DIRECTION
? -500.
                                    A               B
P(VICTORY)                       .64600          .74600
E(BTL RDS)                      1.59400         1.59400
  TRNG HRS                     55.92260        55.92260
  TRNG RDS                     76.25000        76.25000
 TRNG COST                   6373.00000      7373.00000
WHICH DO YOU PREFER.  IF YOU ARE INDIFFERENT TYPE I.
? I
```

Figure 19. Example of Interactive Optimization Computer Program.

```
THE TRADEOFFS ARE
P(VICTORY)                    1.00000
E(BTL RDS)                     .20000
  TRNG HRS                   -.02000
  TRNG RDS                   -.02000
 TRNG COST                   -.00020
 .1547774
 .1797636
 .1547774
 .1797636
NEW DECISION VECTOR
X1          16.00000
X2          15.00000
NEW OPERATING POINT
     .43600
    1.05000
   13.81030
   14.41660
1307.99400
INPUT NUMBER OF POINTS TO SEE IN STEP SIZE
? 7
         .6460       1.5940      55.9226      76.2500    6373.0000

         .6229       1.5239      43.9040      65.9444    5945.4990

         .6096       1.4536      41.8853      55.6339    5017.9980

         .5850       1.3630      34.8667      45.3333    4098.4970

         .5562       1.2722      27.3481      35.0277    3162.9960

         .5232       1.1662      20.8294      24.7222    2235.4950

         .4360     _ 1.0500      13.8103      14.4166    1307.9940

INPUT NUMBER OF PREFFERED POINT
? 3
IF YOU WISH TO TERMINATE TYPE T.  OTHERWISE, TYPE C.
? T
OPTIMAL X
      10.6667       3.3333
    .256 CP SECONDS EXECUTION TIME
```

Figure 19. (Continued)

membered that the original and Fields' solutions were obtained from algorithms designed for, and limited by, a primary and one constraint response function. Their solutions are supposedly precise mathematical programming solutions. In solving these problems with the methodology of this research, close approximation to the previous solutions will be considered validation of the methodology. More precise approximations could have been obtained with numerous iterations of the methodology and extremely large numbers of step-size alternatives to more accurately approach the constraint values. Such a procedure would have approached the numerous iterations of Fields. The solutions obtained in this research are meant to approximate the effort which would be expended by a decision maker. It will be seen that even without extensive computer time or iterations, the methodology of this research compares favorably with the other solution techniques.

The first problem is the Umland and Smith problem shown in Figure 10 and with response functions represented by Equations 2.20 and 2.21. A canonical analysis of Equation 2.20 indicated a stationary point, $\underline{x}_0$ = (2.25,2.35) and eigenvalues $\lambda_1 = -7.38$ and $\lambda_2 = -2.61$. This surface is a maximum. Equation 2.21 has a stationary point $\underline{x}_0$ = (1.15, 0.11) and eigenvalues $\lambda_1 = -10.99$ and $\lambda_2 = -3.39$. This surface is also a maximum. The initial point was chosen to be $\underline{x}_0$ of the primary response. Figures 20, 21, and 22 graph the movement of the algorithm while Table 5 compares results.

The next problem is the first Myers and Carter problem given by Equations 2.24 and 2.25. A canonical analysis of Equation 2.24 yielded $\underline{x}_0$ = (-8.08, 3.89, 3.85), which is outside the constraint region, and

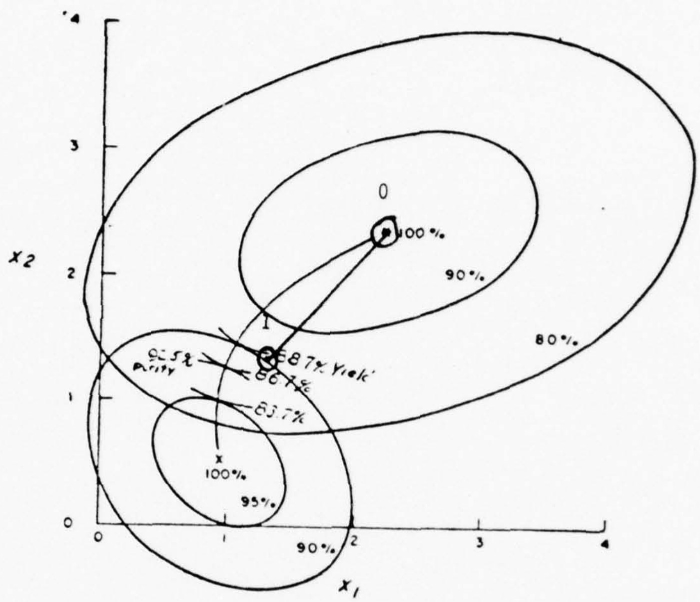

Figure 20. Algorithm Movement on Umland and Smith Problem, $\hat{y}_s \leq 90.0$.

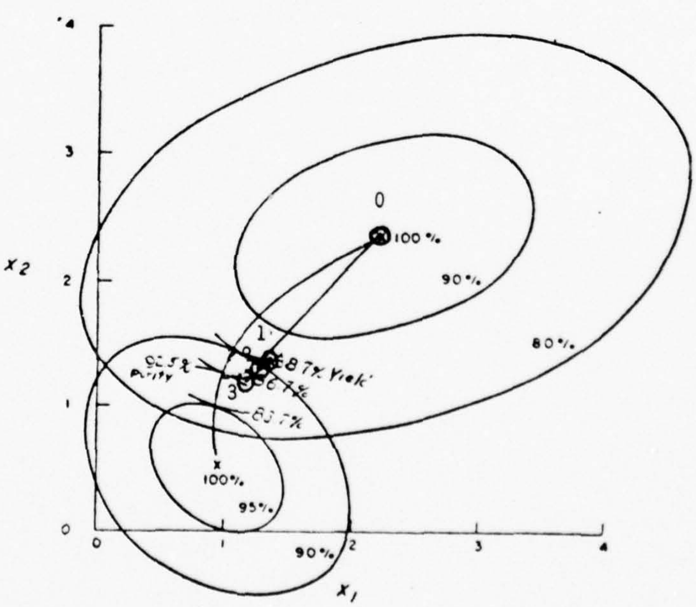

Figure 21. Algorithm Movement on Umland and Smith Problem, $\hat{y}_s \leq 92.5$.

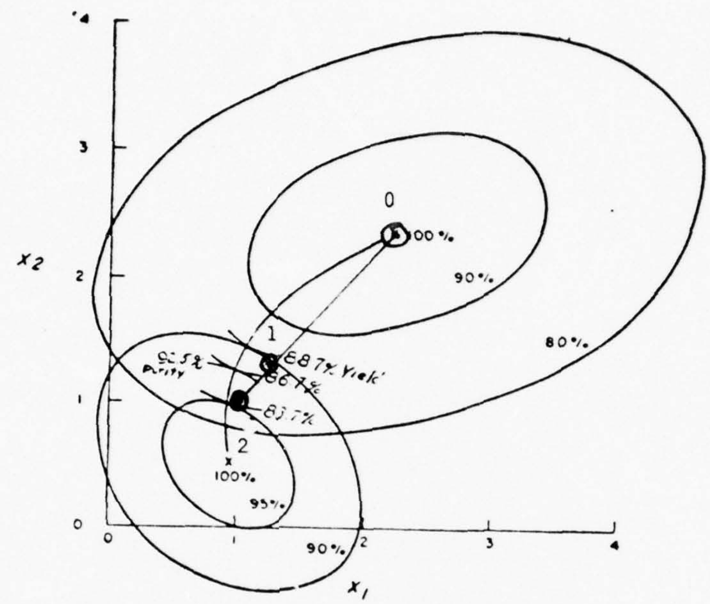

Figure 22. Algorithm Movement on Umland and Smith Problem, $\hat{y}_s \leq 95.0$.

Table 5. Comparison of Umland and Smith Problem Solutions.

Variable	Umland and Smith	Fields	This Research
$\hat{y}_{p1}$	88.68	88.66	88.11
$\hat{y}_{s1}$	89.995	89.9997	90.003
x_{11}	1.075	1.082	1.285
x_{12}	1.479	1.475	1.343
$\hat{y}_{p2}$	86.73	86.64	86.29
$\hat{y}_{s2}$	92.47	92.4998	92.47
x_{12}	1.005	1.0056	1.174
x_{22}	1.316	1.310	1.223
$\hat{y}_{p3}$	83.66	83.47	83.43
$\hat{y}_{s3}$	94.87	94.99	94.99
x_{13}	0.965	0.966	1.013
x_{23}	1.088	1.074	1.058

eigenvalues $\lambda_1 = -25.65$, $\lambda_2 = -2.63$, and $\lambda_3 = 0.18$. This surface is a slight saddle system with an optimum outside the region of experimentation. One must beware of local optima during the optimization procedure. Canonical analysis of Equation 2.25 showed $\underline{x}_0 = (.52,-1.18, .08)$ and eigenvalues $\lambda_1 = 10.55$, $\lambda_2 = 3.56$, and $\lambda_3 = 0.98$, which indicates a minimum surface. The algorithm was initialized at various starting points. Table 6 details the results of these searches and Table 7 compares the optimum solution with previous results. The local optima found in this research were also found in Fields' investigation. This surface also required the use of ridge system procedures during its optimization.

The final problem is the Myers and Carter Problem Two described by Equations 2.26, 2.27, 2.28, and 2.29 and graphed in Figure 14. Canonical analysis of Equation 2.26 indicated $\underline{x}_0 = (-3.72, 4.09)$ and eigenvalues $\lambda_1 = 12.52$ and $\lambda_2 = 1.13$. As seen in the Figure, this system is a minimum with the stationary point outside the region of experimentation. Equation 2.27 has an $\underline{x}_0 = (-.44, -.31)$ and eigenvalues $\lambda_1 = -9.91$ and $\lambda_2 = 2.55$ which indicates a saddle system. The constraint of Equation 2.29, however, is so restrictive that virtually all of the saddle effect is eliminated within the feasible region. It is interesting to note that such a restrictive and arbitrary constraint was necessitated by the Myers and Carter and Fields techniques. Utilizing the methodology of this research, however, a more meaningful constraint such as cost or production time could have been incorporated into the problem formulation.

The constraint formulation of the Myers and Carter Problem Two re-

Table 6. Algorithm Search Results, Myers and Carter Problem One.

Point	x_1	x_2	x_3	$\hat{y}_p$	$\hat{y}_s$
Starting	.52	-1.18	.08		
Solution	1.00	- .06	-.52	70.93	64.08
Starting	2.5	-2.5	-2.5		
Solution	2.02	-1.17	-0.69	73.51	64.70
Starting	2.5	2.5	2.5		
Solution	1.29	-0.30	-0.61	72.09	64.73
Starting	0.	0.	0.		
Solution	1.59	-0.63	-0.64	73.03	64.58

Table 7. Comparison of Myers and Carter Problem One Solutions.

Variable	Myers and Carter	Fields	This Research
$\hat{y}_p$	73.66	73.91	73.51
$\hat{y}_s$	65.22	64.9997	64.70
x_1	2.07	2.13	2.02
x_2	-1.15	-1.25	-1.17
x_3	-0.6	-0.62	-0.69

quired the utilization of the Cyclic Coordinate Penalty Function suboptimization program. The procedure was initialized at $\underline{x}_0$ of $\hat{y}_s$. Figures 23, 24, and 25 trace the iteration solutions of the optimization algorithm. Table 8 compares the results of this research with earlier results.

In the previously solved problems of this section, a close approximation to past results was obtained by the methodology developed in this research. The surfaces optimized were representative of multiple response surface shapes. Two constraint formulations for the feasible region were optimized in two and three variable problems. Application of the adapted Interactive Vector Maximal algorithm to multiple response surfaces has increased the potentiality of their optimization. The restriction of a primary response and one or two constraint responses no longer applies. Theoretically sound optimization may now be performed on large scale multiple response surfaces of various feasible region constraint definitions. In the next chapter, this research will demonstrate the methodology on a training problem applicable to OTEA.

Table 8. Comparison of Myers and Carter Problem Two Solutions.

Variable	Myers and Carter	Fields	This Research
$\hat{y}_p$	67.80	67.57	67.78
$\hat{y}_s$	88.19	86.81	87.996
x_1	.85	.60	.8502
x_2	-.60	-.80	-.5971

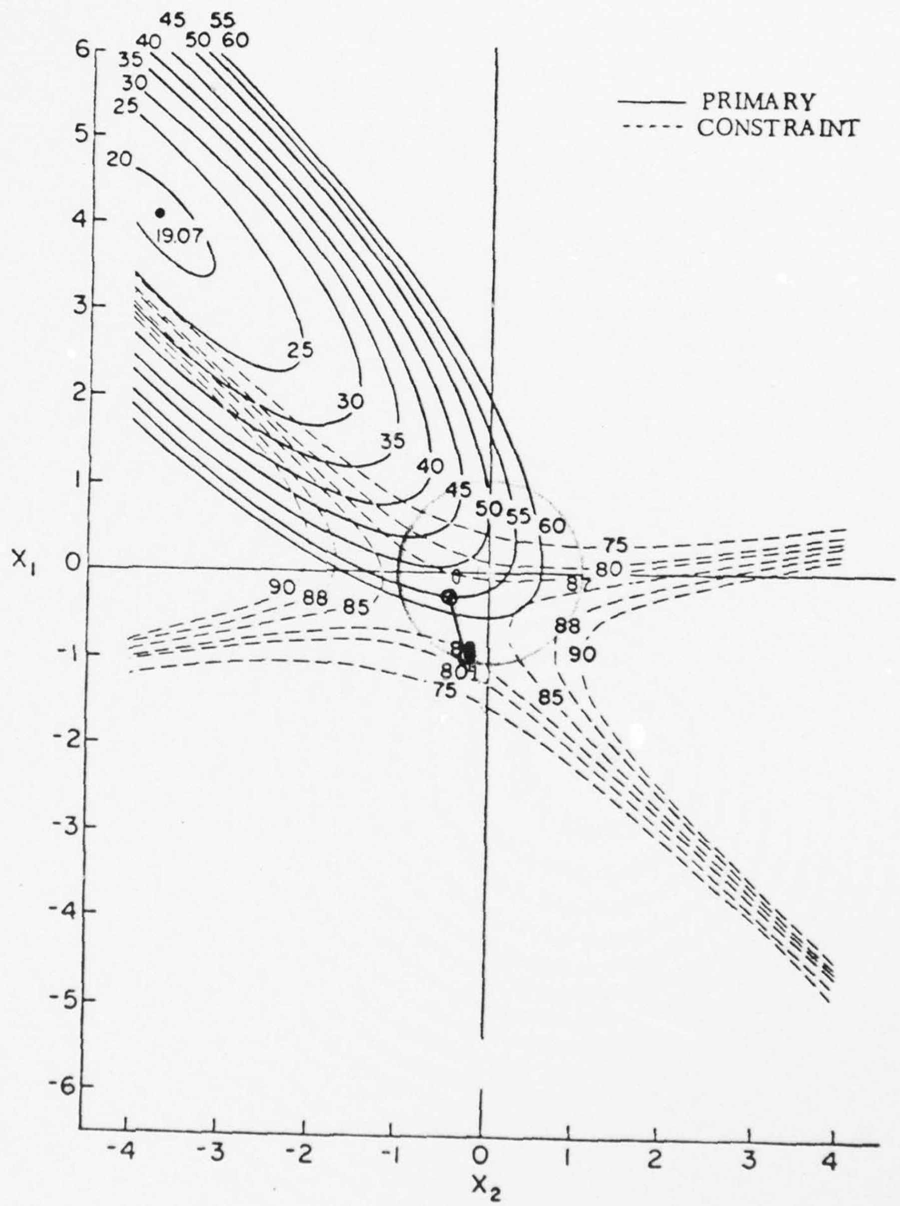

Figure 23. First Iteration of Myers and Carter Problem Two Optimization.

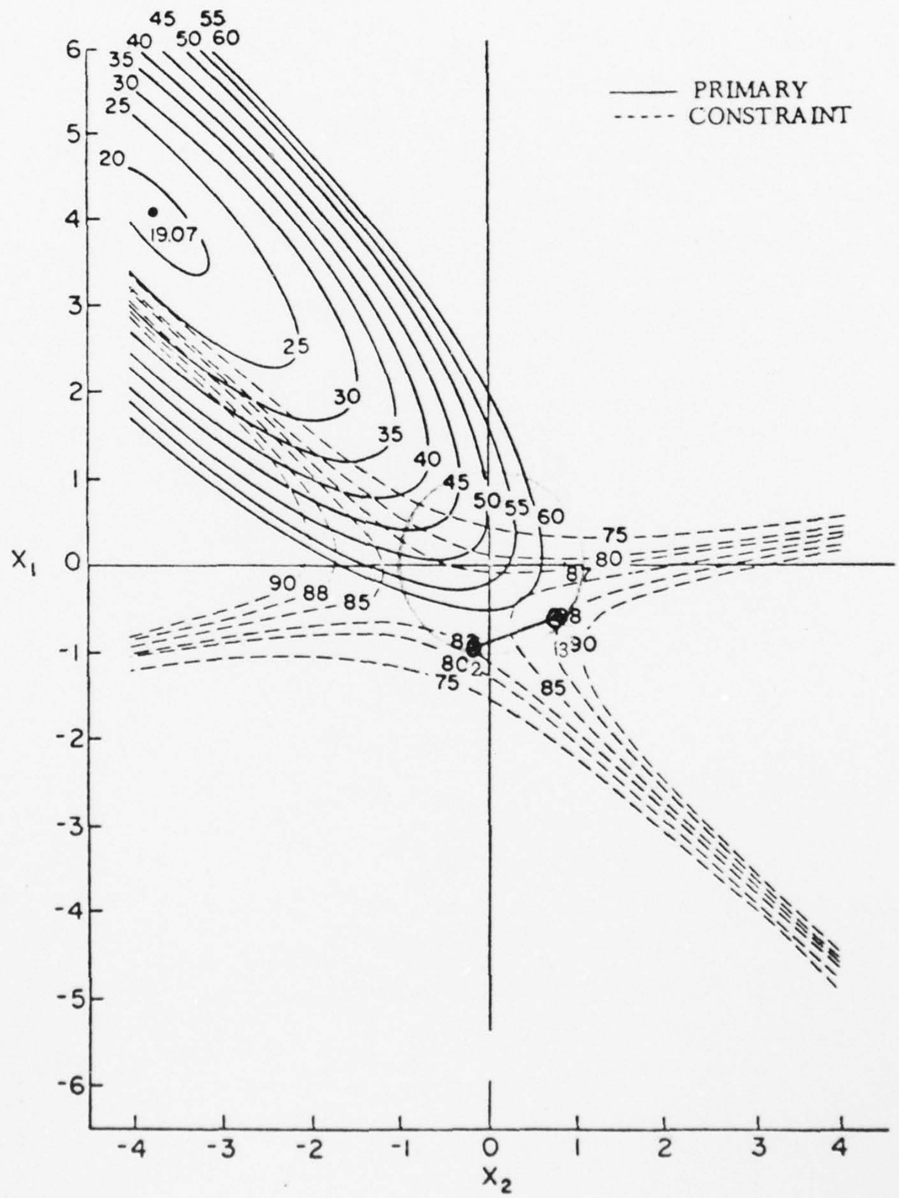

Figure 24. Second and Third Iterations of Myers and Carter Problem Two Optimization.

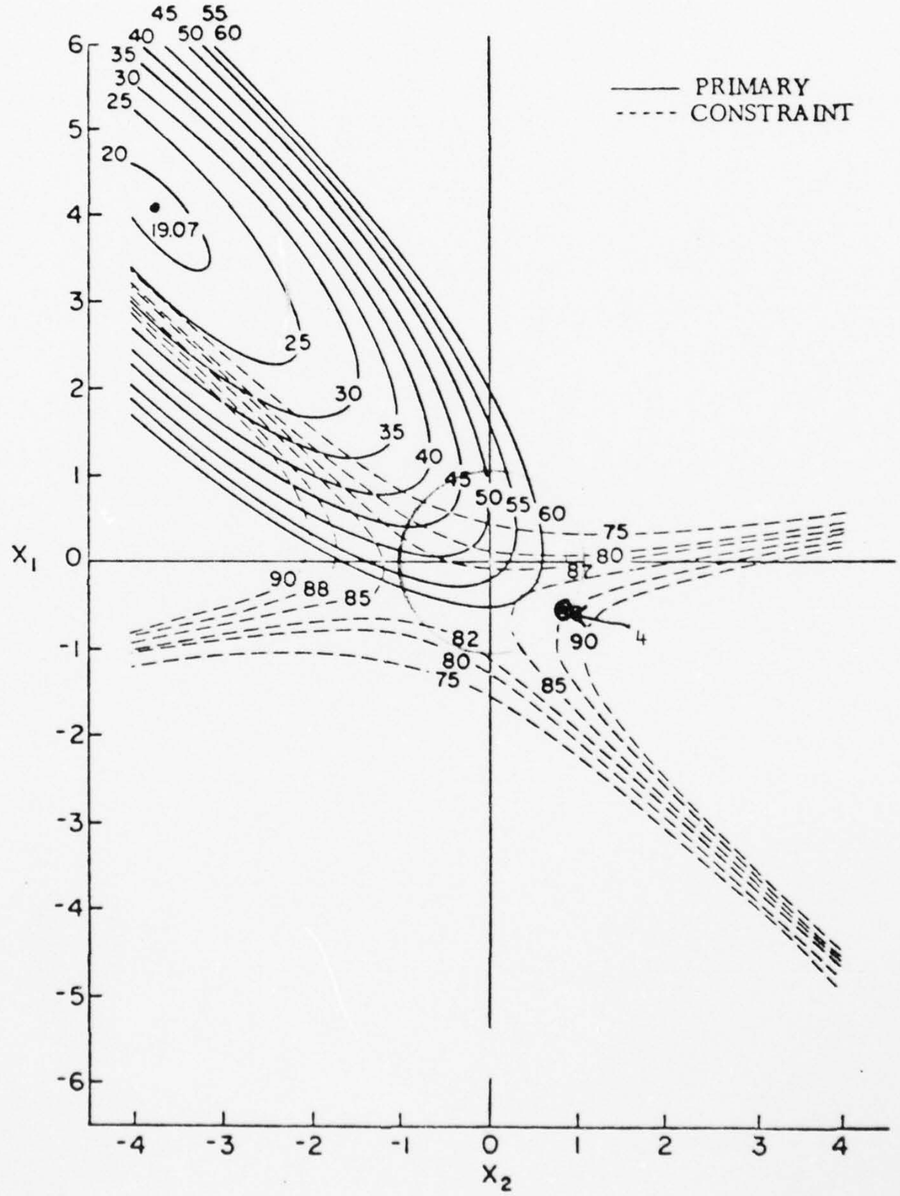

Figure 25. Final Iteration of Myers and Carter Problem Two Optimization.

CHAPTER IV

APPLICATION OF MULTIPLE RESPONSE SURFACE OPTIMIZATION TO AN OPERATIONAL TEST PROBLEM

Introduction to the Problem

In Chapter I the importance and effects of training in operational testing was discussed. The utilization of computer simulation concurrent with an OT was also discussed. In Chapter III a methodology was developed to analyze and optimize multiple response surfaces. The role of the decision maker in the interactive algorithm and the benefits accrued by his participation were discussed. In this chapter, computer simulation and the methodology of this research will be applied to a hypothetical acquisition program.

Subsequent to the cancellation of the costly Main Battle Tank 1970 (MBT70) acquisition program, the Army began development of the less costly MBT76. As one means of cost reduction, all factors of system effectiveness were considered rather than exclusive consideration of the MBT76 technological capabilities. The Project Manager (PM) felt that crew training could be of utmost importance in overall MBT76 combat effectiveness. Prior to OT II, he directed an analysis of the effects of crew training utilizing a computer simulation of a combat situation indicative of the European environment. The laser ranging and optical tracking of the MBT76 were sophisticated enough to negate any effect of training on weapon accuracy. Consequently the PM directed that mean time to fire the first round, mean time between rounds, and probability

of sensing be studied as system factors affected by crew training. In this initial stage, he also directed that one scenario, an engagement between two tanks in the open at a range of 1000 meters, be analyzed to establish feasibility of the methodology. This scenario was representative of tank combat in the European theater.

Utilization of the AMSAA Tank Duel Simulation

The MBT76 Analysis Team (AT) used the AMSAA Tank Duel simulation programmed by Mr. Robert Lake. It is a low level, small scale, two-sided, deterministic model used to simulate brief fire engagements between two armored vehicles. The model plays a defending vehicle (MBT76, Blue) which is stationary and fires first at an attacker (Red) which is fully exposed. The engagement ends when a kill occurs or when a time limit expires. It is programmed in FORTRAN IV for the BRLESC computer. Inputs include various probabilities of hit and kill, expected time to fire rounds, and probabilities of sensing. Outputs include the probability of victory and expected number of rounds fired.

The AMSAA Tank Duel Model was well suited to the AT's needs with a few modifications. Planning to use statistical analysis, the AT required a stochastic simulation. Where the model utilized the mean of certain probability distributions, the AT decided to input random deviates from the distributions. It was assumed that the random variables in this model were normally distributed. The means and variances of the various inputs, shown in Table 9, were based on OT I and DT I results for the MBT76 and best intelligence estimates for the Red. After converting the model for use on the CDC CYBER 74 computer, as shown in

Appendix B, the AT wrote two programs to generate the random deviates.

The first program, listed in Appendix A, utilized a CDC internal random number generator to generate 200 Uniform (0,1) random deviates. The generator was analyzed by a Chi-square test which showed that at $\alpha = .11$ the random deviates were U(0,1). Table 10 shows the distribution of the deviates. The Chi-square statistic was computed as follows (33)

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \tag{4.1}$$

to be $\chi_0^2 = 11.0$. The U(0,1) deviates were then converted to N(0,1) deviates and subsequently to normal random deviates of the distributions in Table 9. This conversion was accomplished by the well known and tested Fishman Equations (24),

$$\begin{aligned} X_1 &= (-2 \log U_1)^{1/2} \cos 2\pi U_2 \\ X_2 &= (-2 \log U_1)^{1/2} \sin 2\pi U_2, \end{aligned} \tag{4.2}$$

where X_i are N(0,1) and U_i are U(0,1). This conversion was accomplished by a computer program listed in Appendix A. A Chi-square statistic of $\chi_0^2 = 5.28$ was computed for the N(0,1) deviates as shown in Table 11. At $\alpha = .27$ the deviates are distributed N(0,1).

Specification of the scenario by the PM allowed certain model parameters to be fixed for all trials of the model. These values are shown in Table 12. The time of flight was based on use of High Explosive Anti-Tank (HEAT) rounds with a muzzle velocity of 3800 feet per

Table 9. Input Variable Normal Distributions

Input Variable	BLUE Mean	BLUE Variance	RED Mean	RED Variance
P(Hit 1st Rd)	.75	.0025	.60	.0025
P(Rehit)	.85	.0011	.75	.0011
P(Hit|Sensing 1st Rd Miss)	.80	.0011	.7	.0011
P(Hit|Loss of 1st Rd Miss)	.775	.0017	.625	.0017
P(Kill|1st Rd Hit)	.5	.0011	.45	.0011
P(Kill|Rehit)	.85	.0003	.8	.0003
P(Kill|Hit ∩ Sensing 1st Rd Miss)	.5	.0011	.45	.0011
P(Kill|Hit ∩ Loss of 1st Rd Miss)	.5	.0011	.45	.0011
P(Sensing)			.525	.0006
Time to Fire 1st Rd (sec)			8.5	.6944
Time to Fire Subsequent Rd (sec)			10.5	.6944

Table 10. Distribution of U(0,1) Deviates.

Interval	Observed	Expected
.00 - .05	7	10
.05 - .10	9	10
.10 - .15	10	10
.15 - .20	7	10
.20 - .25	9	10
.25 - .30	12	10
.30 - .35	9	10
.35 - .40	11	10
.40 - .45	6	10
.45 - .50	11	10
.50 - .55	15	10
.55 - .60	11	10
.60 - .65	11	10
.65 - .70	8	10
.70 - .75	9	10
.75 - .80	8	10
.80 - .85	11	10
.85 - .90	15	10
.90 - .95	12	10
.95 -1.00	9	10

Table 11. Distribution of N(0,1) Deviates.

Interval	Observed	Expected
- ∞, - 2.0	2	13.36
-2.0, - 1.5	10	
-1.5, - 1.0	17	18.38
-1.0, - 0.5	33	29.96
-0.5, 0.0	47	38.3
0.0, 0.5	35	38.3
0.5, 1.0	30	29.96
1.0, 1.5	13	18.38
1.5, 2.0	7	8.82
2.0, + ∞	6	4.54

Table 12. Fixed Input Variable Values.

Input Variable	Value
Engagement Time (sec)	120.0
Blue Time of Flight (sec)	.86
Blue Fixed Time to Fire (sec)	7.0
Range (meters)	1000
Blue Rd Reliability	.85
Red Time of Flight (sec)	1.17
Red Fixed Time to Fire (sec)	7.0
Red Rd Reliability	.825

second for the MBT76 and 2800 feet per second for the Red tank. The fixed time to fire variable accounts for the mechanical actions between rounds such as recoil and breech operation. Thus the firing times analyzed by the AT in this demonstration are human actions such as issuing a fire order, loading the round, and tracking the target. A sample of the model output is shown in Figure 26.

Derivation of Multiple Response Surfaces

The modified AMSAA Tank Duel Model could now be utilized by the AT for the derivation of multiple response surfaces. As directed by the PM, mean time to fire the first round (ξ_1), mean time between rounds (ξ_2), and probability of sensing (ξ_3) were chosen as independent design variables while probability of an MBT76 victory ($\hat{y}_1$) and expected number of MBT76 rounds fired ($\hat{y}_2$) were chosen as the response variables. Based on experience by OTEA and TRADOC in crew performance, realistic ranges were chosen for the design variables. Mean time to fire the first round, human action component, ranged between 30 and 8 seconds. Mean time between rounds, human component, ranged between 30 and 5 seconds. Probability of sensing ranged between .0 and .6. The Red probability of sensing is somewhat higher since the Red round has a lower muzzle velocity and, consequently, is easier to sense.

A full 2^3 experimental design was performed on the AMSAA Model. Table 13 details the design and the responses. The values in parentheses are the ξ_i (natural) independent variable values while those outside the parentheses are the coded values as defined by Equation 2.4. Next the AT performed multiple linear regression on this data using the Statisti-

```
         A MEETING ENGAGEMENT BETWEENBLUE       ANDRED
         THE TIME LIMIT IS  120.00SECONDS
         RANGE IS 1000 METERS
         BLUE DATA IS BLU BLU BLU
         RED DATA IS RED RED RED
        TFL     TT      T1     TS   PH1  PHH  PHS  PHL  KH1  KHH  KHS  KHL   PS   REL
BLUE    .86   7.00 25.00 25.00 .780 .811 .764 .867 .496 .849 .504 .541 .100  .85
RED    1.17   7.00  8.44 10.34 .604 .762 .744 .562 .453 .795 .463 .462 .498  .83
         PROB(BLUE   WINS)=   .336
         PROB(RED    WINS)=   .653
         PROB(NO DECISION)=    .012
         E(RDS FORBLUE )=      .673
         E(RDS FOR RED  )=     2.360
```

Figure 26. Sample Output From AMSAA Tank Duel Model.

cal Package for the Social Sciences (SPSS) regression computer program (49) discussed in Appendix C. Figure 27 is the output from the SPSS program on the data of Table 13. The top half of the Figure concerns $\hat{y}_1$ while the bottom half concerns $\hat{y}_2$. In the upper right quadrant of each half is the ANOVA table for regression and residual error. The lower left quadrant contains the regression coefficients of the independent variables. The following two response equations are determined from Figure 27,

$$\begin{aligned} \hat{y}_1 &= -.037x_1 - .023x_2 + .002x_3 + .344 \\ \hat{y}_2 &= -.074x_1 - .054x_2 + .010x_3 + .697 \end{aligned} \tag{4.3}$$

where $\hat{y}_1$ is probability of victory, $\hat{y}_2$ is expected number of rounds fired, x_1 is time to fire the first round, x_2 is time between rounds, and x_3 is probability of sensing. An interesting result is that probability of sensing over the region of experimentation is statistically insignificant.

The AT performed two further statistical tests on the data of Table 13. First a goodness of fit test was computed (33). The residual sum of squares is separated into two parts, a sum of squares due to pure experimental error and sum of squares due to lack-of-fit,

$$SS_E = SS_{PE} + SS_{LOF} \quad . \tag{4.5}$$

Sum of square pure error is calculated by

Table 13. 2^3 Design Variable Values, First Design.

x_1		x_2		x_3		y_1	y_2
-1	(20)	-1	(20)	-1	(.0)	.407	.795
1	(30)	-1	(20)	-1	(.0)	.341	.738
-1	(20)	1	(30)	-1	(.0)	.347	.709
1	(30)	1	(30)	-1	(.0)	.307	.581
-1	(20)	-1	(20)	1	(.2)	.450	.931
1	(30)	-1	(20)	1	(.2)	.304	.612
-1	(20)	1	(30)	1	(.2)	.356	.721
1	(30)	1	(30)	1	(.2)	.310	.637
0	(25)	0	(25)	0	(.1)	.318	.629
0	(25)	0	(25)	0	(.1)	.301	.576
0	(25)	0	(25)	0	(.1)	.342	.729
0	(25)	0	(25)	0	(.1)	.329	.739
0	(25)	0	(25)	0	(.1)	.371	.690
0	(25)	0	(25)	0	(.1)	.336	.673

```
DEF. VAR... PV

FINAL STEP.

MULTIPLE R      .8144 ANOVA         DF   SUM SQUARES  MEAN SQ.       F
R SQUARE        .6633 REGRESSION     3.          .015       .005     6.567
STD DEV         .0279 RESIDUAL      10.          .008       .001 SIG  .010

VARIABLE          B        S.E. B       F      SIG.        BETA   ELASTICITY

TB1             -.037       .010    14.311   .004      -.69414        0
TBS             -.025       .010     5.338   .043      -.42394        0
PS               .002       .010      .052   .824       .04193        0
CONSTANT         .344       .007  2138.455   .000

ALL VARIABLES ARE IN THE EQUATION.

DEF. VAR... ER

FINAL STEP.

MULTIPLE R      .7669 ANOVA         DF   SUM SQUARES  MEAN SQ.       F
R SQUARE        .5881 REGRESSION     3.          .067       .022     4.759
STD DEV         .0684 RESIDUAL      10.          .047       .005 SIG  .026

VARIABLE          B        S.E. B       F      SIG.        BETA   ELASTICITY

TB1             -.074       .024     9.226   .013      -.61647        0
TBS             -.054       .024     4.888   .051      -.44873        0
PS               .010       .024      .162   .695       .08178        0
CONSTANT         .697       .018  1452.529   .000
```

Figure 27. SPSS Multiple Linear Regression of First Design.

$$SS_{PE} = \sum_{i=1}^{n_2} y_{ci}^2 - \frac{\sum_{i=1}^{n_2} y_{ci}}{n_2} \tag{4.6}$$

where y_{ci} are observations at the center point and n_2 is the number of center points. Since residual sum of squares is given by SPSS and sum of squares pure error is computed by Equation 4.6, sum of squares lack-of fit can be computed from 4.5. An F test statistic is then computed by

$$F_0 = \frac{SS_{LOF}/(n-p-n_e-1)}{SS_{PE}/n_e} \sim F(n-p-n_e-1),n_e \tag{4.7}$$

where n is total number of observations, p is the number of variables, and

$$n_e = \sum_{i=1}^{m} (p_i - 1) \tag{4.8}$$

where m is the number of different variable levels and p_i is the number of observations at each level. For the first 2^3 design $F_{0_{\hat{y}1}} = 1.86$ and and $F_{0_{\hat{y}2}} = 1.46$, neither of which are significant at the $\alpha = .10$ level. Therefore the fit of Equations 4.3 and 4.4 is satisfactory.

The final test was to establish a confidence interval about the mean predicted responses at the center point of the design. The desired confidence intervals are computed by (33)

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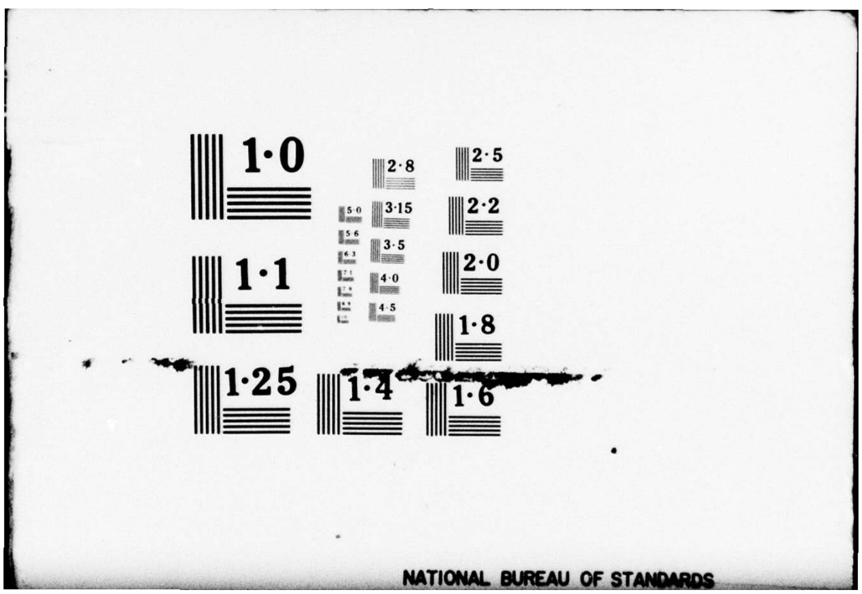
1·0
2·8
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4·5
1·8
1·25
1·4
1·6
NATIONAL BUREAU OF STANDARDS

$$\bar{y}_c - t_{\alpha/2,n_2-1}\frac{S}{\sqrt{n_2}} \le \mu \le \bar{y}_c + t_{\alpha/2,n_2-1}\frac{S}{\sqrt{n_2}} \qquad (4.9)$$

where

$$S^2 = \sum_{i=1}^{n_2} (y_{ci}-\bar{y}_c)^2/(n_2-1) \qquad (4.10)$$

and

$$\bar{y}_c = \sum_{i=1}^{n_2} y_{ci}/n_2 \quad . \qquad (4.11)$$

The following are 90% confidence intervals for the values of the mean predicted responses at the center point of the first design:

$$\text{Probability of Victory;} \quad .314 \le \mu y_1 \le .352 \qquad (4.12)$$

$$\text{Expected Number of Rounds;} \quad .622 \le \mu y_2 \le .724 \quad . \qquad (4.13)$$

Next the AT performed a steepest ascent analysis, starting from the center point, and proceeding in directions determined by Equations 4.3 and 4.4. Table 14 shows the results of this optimization. The new center point for the next 2^3 design is $\xi_1 = 10.0$, $\xi_2 = 15.67 \simeq 16.0$, and $\xi_3 = .115$. Table 15 and Figure 28 show the results of this second design. The fitted response equations for this design are

$$\hat{y}_1 = -.025x_1 - .032x_2 - .010x_3 + .525 \qquad (4.14)$$

$$\hat{y}_2 = -.043x_1 - .112x_2 - .031x_3 + 1.158 \quad . \qquad (4.15)$$

Table 14. Steepest Ascent Optimization From First Center Point.

MOVE	ξ_1	ξ_2	ξ_3	y_1
Δ	-1.0	-.622	.001	
Base	25.0	25.0	.1	.333
+5 Δ	20.0	21.89	.105	.357
+10Δ	15.0	18.78	.11	.425
+15Δ	10.0	15.67	.115	.601
+16Δ	9.0	15.05	.116	.557
+17Δ	8.0	14.43	.117	.549

Table 15. 2^3 Design Variable Values, Second Design.

x_1		x_2		x_3		y_1	y_2
-1	(8)	-1	(11)	-1	(0)	.593	1.190
1	(12)	-1	(11)	-1	(0)	.491	1.146
-1	(8)	1	(21)	-1	(0)	.520	1.093
1	(12)	1	(21)	-1	(0)	.528	1.082
-1	(8)	-1	(11)	1	(.24)	.610	1.499
1	(12)	-1	(11)	1	(.24)	.535	1.251
-1	(8)	1	(21)	1	(.24)	.480	1.028
1	(12)	1	(21)	1	(.24)	.438	0.984
0	(10)	0	(16)	0	(.12)	.577	1.186
0	(10)	0	(16)	0	(.12)	.528	1.107
0	(10)	0	(16)	0	(.12)	.510	1.168
0	(10)	0	(16)	0	(.12)	.514	1.206
0	(10)	0	(16)	0	(.12)	.492	1.110
0	(10)	0	(16)	0	(.12)	.518	1.163

```
DEF. VAR... PV

FINAL STEP.

MULTIPLE R     .7146 ANOVA            DF   SUM SQUARES   MEAN SQ.       F
R SQUARE       .5106 REGRESSION        3.          .014       .005   3.478
STD DEV        .0364 RESIDUAL         10.          .013       .001 SIG  .058

VARIABLE         B        S.E. B         F       SIG.       BETA   ELASTICITY

TB1           -.025         .013     3.810    .080     -.43178            0
TBS           -.032         .013     6.036    .034     -.54349            0
PS            -.010         .013      .589    .461     -.16971            0
CONSTANT       .525         .010  2906.184    .000

ALL VARIABLES ARE IN THE EQUATION.

DEF. VAR... ER

FINAL STEP.

MULTIPLE R     .8048 ANOVA            DF   SUM SQUARES   MEAN SQ.       F
R SQUARE       .6476 REGRESSION        3.          .124       .041   6.126
STD DEV        .0821 RESIDUAL         10.          .067       .007 SIG  .012

VARIABLE         B        S.E. B         F       SIG.       BETA   ELASTICITY

TB1           -.043         .029     2.232    .166     -.28043            0
TBS           -.112         .029    14.980    .003     -.72653            0
PS             .031         .029     1.168    .305      .20285            0
CONSTANT      1.158         .022  2784.012    .000
```

Figure 28. SPSS Multiple Linear Regression of Second Design.

Again it is noted that probability of sensing is not statistically significant. Goodness of fit computations for this data are $F_{0_{\hat{y}1}} = 2.11$ which is not significant at $\alpha = .10$ and $F_{0_{\hat{y}2}} = 7.27$ which is not significant at $\alpha = .15$. The 90% confidence intervals for the responses at the center point are:

$$\text{Probability of Victory;} \quad .499 \leq \mu y_1 \leq .548 \tag{4.16}$$

$$\text{Expected Number of Rounds;} \quad 1.124 \leq \mu y_2 \leq 1.190 \tag{4.17}$$

Upon determining the path of steepest ascent, a change in ξ_3, the probability of sensing, in a negative direction was noted. Since ξ_3 has been statistically insignificant and clearly does not improve in the negative direction, no change in x_3 was made in the initial steepest ascent optimization. Table 16 displays the results of this search. ξ_1 and ξ_2 have now reached the lower bound of their practical ranges. From this point a uni-direction search was made along the ξ_3 direction to determine if any further improvement could be obtained. Table 17 shows the results of this uni-direction search. Based on this search and the fact that ξ_3 has been insignificant in two successive 2^3 designs, the AT decided to eliminate ξ_3 from further designs as statistically insignificant and fix it at .3, the median of its practical range. Apparently, at the given range and with the given probabilities of hit and kill, the ability to sense a round is not critical. The engagement seems to be won on the speed of firing the first round and a second round if required. Given another scenario, it is not unreasonable to expect that ξ_3 would be significant. The center point is moved to

Table 16. Steepest Ascent Optimization From Second Center Point

Move	ξ_1	ξ_2	ξ_3	y_1
Δ	-.31	-1.0	.00	
Base	10	16	.12	.523
5Δ	9.69	11	.12	.572
8Δ	9.39	8	.12	.620
9Δ	9.07	7	.12	.650
10Δ	8.76	6	.12	.665
11Δ	8.45	5	.12	.671

Table 17. Uni-direction Search Along ξ_3.

ξ_1	ξ_2	ξ_3	y_1	y_2
8	5	.2	.661	1.648
8	5	.3	.696	1.624
8	5	.4	.693	1.737
8	5	.5	.673	1.650
8	5	.55	.650	1.637
8	5	.6	.658	1.596

$\xi_1 = 12.0$ and $\xi_2 = 10.0$.

For the third 2^3 design, the design variable ranges were chosen so as to border on the optimum lower bound and include a large portion of the region of experimentation. Table 18 and Figure 29 show the third design and its results. The response equations are

$$\hat{y}_1 = -.042x_1 - .063x_2 + .575 \tag{4.18}$$

$$\hat{y}_2 = -.133x_1 - .174x_2 + 1.339 \quad . \tag{4.19}$$

The $F_{0_{\hat{y}1}} = 2.99$ and $F_{0_{\hat{y}2}} = 4.23$ are not significant at $\alpha = .10$ which justifies elimination of ξ_3 as a design variable. The 90% confidence intervals at the center point are:

$$\text{Probability of Victory;} \quad .572 \leq \mu y_1 \leq .598 \tag{4.20}$$

$$\text{Expected Number of Rounds;} \quad 1.332 \leq \mu y_2 \leq 1.404 \tag{4.21}$$

Since the design now bordered on the lower bound of the practical region, a second order design was employed to determine if the fit could be improved with the use of second order equations. To create a Uniform Precision Rotatable Central Composite Design (UP CCD), axial points were added as shown in Table 19 and a second order polynomial was fit using polynomial regression, as shown in Figure 30. The goodness of fit test revealed $F_{0_{\hat{y}1}} = 2.66$ and $F_{0_{\hat{y}2}} = .60$, both of which are improvements over the linear model. Thus the second order response equations were adopted:

Table 18. 2^3 Design Variable Values, Third Design.

x_1		x_2		y_1	y_2
-1	(8)	-1	(5)	.669	1.635
1	(16)	-1	(5)	.581	1.315
-1	(8)	1	(15)	.538	1.235
1	(16)	1	(15)	.460	1.021
0	(12)	0	(10)	.577	1.337
0	(12)	0	(10)	.585	1.380
0	(12)	0	(10)	.581	1.366
0	(12)	0	(10)	.573	1.332
0	(12)	0	(10)	.609	1.426

Table 19. Axial Points Added to the Third Design.

x_1		x_2		y_1	y_2
-1.414	(6.344)	0	(10)	.591	1.404
1.414	(17.656)	0	(10)	.518	1.148
0	(12)	-1.414	(2.93)	.617	1.504
0	(12)	1.414	(17.07)	.533	1.092

```
DEP. VAR... PV

FINAL STEP.

MULTIPLE R     .9588 ANOVA            DF   SUM SQUARES  MEAN SQ.       F
R SQUARE       .9192 REGRESSION        2.         .023       .011   34.138
STD DEV        .0133 RESIDUAL          6.         .002       .000 SIG  .001

VARIABLE          B       S.E. B        F      SIG.       BETA   ELASTICITY

TB1            -.042        .009     20.661  .004      -.52742          0
TBS            -.063        .009     47.615  .000      -.80066          0
CONSTANT        .575        .006   8917.499  .000

ALL VARIABLES ARE IN THE EQUATION.

DEP. VAR... EP

FINAL STEP.

MULTIPLE R     .9551 ANOVA            DF   SUM SQUARES  MEAN SQ.       F
R SQUARE       .9121 REGRESSION        2.         .192       .096   31.146
STD DEV        .0555 RESIDUAL          6.         .018       .003 SIG  .001

VARIABLE          B       S.E. B        F      SIG.       BETA   ELASTICITY

TB1            -.133        .028     23.166  .003      -.58242          0
TBS            -.174        .028     39.127  .001      -.75692          0
CONSTANT       1.339        .018   5240.051  .000
```

Figure 29. SPSS Multiple Linear Regression of Third Design.

DEP. VAR... PV

FINAL STEP.

MULTIPLE R	.9427	ANOVA	DF	SUM SQUARES	MEAN SQ.	F
R SQUARE	.8887	REGRESSION	5.	.028	.006	11.176
STD DEV	.0224	RESIDUAL	7.	.004	.001	SIG .003

VARIABLE	B	S.E. B	F	SIG.	BETA	ELASTICITY
TB12	-.015	.009	3.510	.103	-.23829	-.01716
TB1TBS	.003	.011	.050	.830	.02810	0
TB1	-.034	.008	17.993	.004	-.53501	0
TBS	-.046	.008	34.138	.001	-.73683	0
TBS2	-.006	.009	.447	.525	-.08502	-.00612
CONSTANT	.585	.010	3398.779	0		

ALL VARIABLES ARE IN THE EQUATION.

DEP. VAR... ER

FINAL STEP.

MULTIPLE R	.9828	ANOVA	DF	SUM SQUARES	MEAN SQ.	F
R SQUARE	.9659	REGRESSION	5.	.324	.065	39.708
STD DEV	.0404	RESIDUAL	7.	.011	.002	SIG .000

VARIABLE	B	S.E. B	F	SIG.	BETA	ELASTICITY
TB12	-.042	.015	7.684	.028	-.19502	-.01976
TB1TBS	.027	.020	1.720	.231	.09148	0
TB1	-.112	.014	61.459	.000	-.54682	0
TBS	-.150	.014	124.760	.000	-.77910	0
TBS2	-.031	.015	4.219	.079	-.14450	-.01464
CONSTANT	1.358	.013	5731.665	0		

Figure 30. SPSS Multiple Polynomial Regression of Second Order Design.

$$\hat{y}_1 = -.016x_1^2 - .006x_2^2 + .003x_1x_2 - .034x_1 - .046x_2 + .585 \tag{4.22}$$

$$\hat{y}_2 = -.042x_1^2 - .031x_2^2 + .027x_1x_2 - .112x_1 - .160x_2 + 1.368 \tag{4.23}$$

To be meaningful for future analysis, the coded variables in Equations 4.22 and 4.23 were transformed to natural design variables

$$\hat{y}_1 = -.001\xi_1^2 - .00024\xi_2^2 + .00015\xi_1\xi_2 + .014\xi_1 - .0062\xi_2 + .629 \tag{4.24}$$

$$\hat{y}_2 = -.0002625\xi_1^2 - .00124\xi_2^2 + .00135\xi_1\xi_2 + .0215\xi_1 - .0234\xi_2 + 1.684\ . \tag{4.25}$$

A canonical analysis was performed with the assistance of the XEIGEN library computer program. Equation 4.22 has a stationary point $\underline{\xi}_0 = (6.176, -10.99)$ outside the region of experimentation, and eigenvalues $\lambda_1 = -.016$ and $\lambda_2 = -.006$ indicating a maximum surface. Equation 4.23 has $\underline{\xi}_0 = (-7.29, -21.18)$ outside the region of experimentation, and eigenvalues $\lambda_1 = -.061$ and $\lambda_2 = -.012$ indicating another maximum surface.

Response equations relating the design variables to training were sought from TRADOC training studies on armored crew training.[1] The approximating relationship between ξ_1, ξ_2 and hours of dry (no live firing) training ($\hat{y}_3$), in the region of experimentation for Equations 4.24 and 4.25, was found to be

---

[1] See Appendix E for an explanation of the derivation of Equations 4.26, 4.27, and 4.28

$$\hat{y}_3 = -2.5556\xi_1 - 2.1667\xi_2 + 87.2009 \quad . \qquad (4.26)$$

The approximating equation for live training rounds fired $(\hat{y}_4)$, in the region of experimentation for Equations 4.24 and 4.25, was found to be

$$\hat{y}_4 = -2.611\xi_1 - 2.9167\xi_2 + 107.30015 \quad . \qquad (4.27)$$

The cost of training $(\hat{y}_5)$, in the region of experimentation for Equations 4.24 and 4.25, based mainly on cost of rounds and of Petroleum, Oil, and Lubricants (POL), was computed to be approximately

$$\hat{y}_5 = -234.999\xi_1 - 262.503\xi_2 + 9667.5135 \quad . \qquad (4.28)$$

Application of the Optimization Methodology to the Derived Multiple Response Surfaces

With the five multiple response surfaces derived in the last section, the AT was prepared to present the PM with optimization and analysis of training effects. The independent variables for his given scenario were mean time to fire first round and mean time between rounds. The response variables were probability of victory for the MBT76, expected number of rounds fired, hours of dry training, live training rounds fired, and cost of training. Foreseeing minimal information gain by its continued inclusion, the PM directed that expected number of rounds fired be eliminated from the optimization. Figure 31 graphs the response surfaces in the area of the region of experimentation.

To acquaint the PM and themselves with the surface, and to alleviate the PM's concern about convergence of the methodology, the AT began a sample optimization with an impractical point, $\xi_1 = 5.0$ and

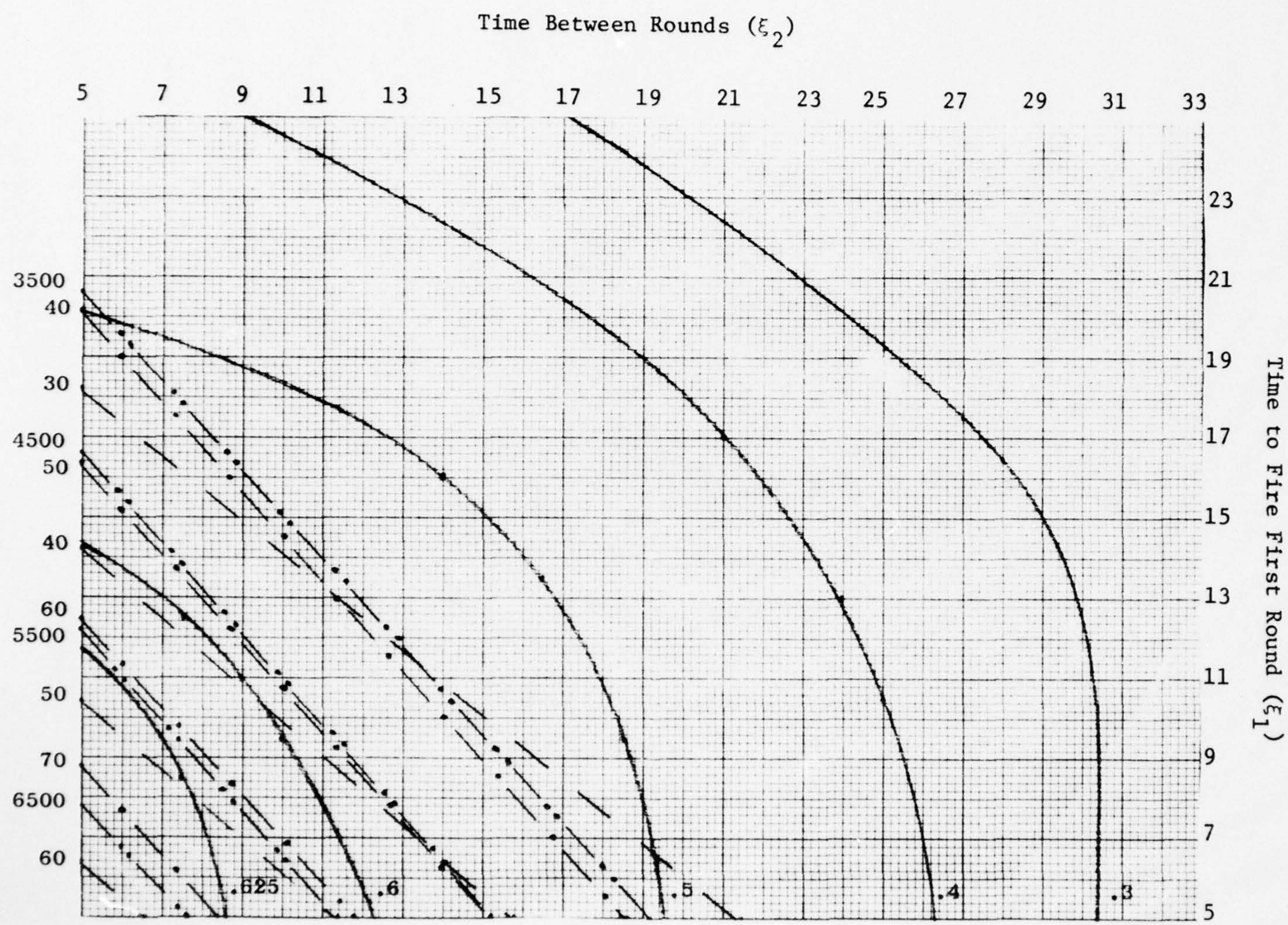

Figure 31 Derived Multiple Response Surfaces.

Figure 31. (Continued). Glossary for Figures 31, 32, 33 and 34.

————————	$\hat{y}_1$	Probability of Victory
—— —— ——	$\hat{y}_3$	Training Hours
—— · —— · ——	$\hat{y}_4$	Training Rounds
—— ·· —— ·· —	$\hat{y}_5$	Training Cost in Dollars

$\xi_2 = 30.0$. The objective was to maximize $\hat{y}_1$ while constraining $\hat{y}_5$ to be less than \$4500.00. Figure 32 depicts the operation of the methodology. It was discovered that larger violations of the constraint on each iteration hastened convergence. The optimum point reached was $\xi_1 = 11.3444$ secs and $\xi_2 = 9.6965$ secs where $\hat{y}_1 = .5929$, $\hat{y}_3 = 37.1966$ hrs., $\hat{y}_4 = 49.3968$ rds, and $\hat{y}_5 = \$4456.22$. A validation was run, as graphed in Figure 33, by moving from the initial point to the region of experimentation optimum and then back to a constrained optimum. This optimum point, which violated the constraint by \$78.16 (1.7%) was $\xi_1 = 11.3684$ secs and $\xi_2 = 9.2105$ secs. Thus the zig-zag behavior of the PM had converged to the optimum constrained point. The small discrepancy was caused by the step-size intervals which were not small enough to permit the constraint to be satisfied exactly.

Analysis of data from the training program prior to OT I and from OT I indicated initial crew performance on the MBT76 to be 30 secs mean time to fire the first round and 25 secs mean time between rounds. Allowing for 7 secs mechanical fixed time this converted to $\xi_1 = 23.0$ secs and $\xi_2 = 18.0$ secs. Performing iterations at this level on the AMSAA simulation, the AT obtained the data in Table 20 and a 90% confidence interval about the probability of victory of

$$.3520 \leq \mu y_1 \leq .4332 \tag{4.29}$$

In an effort to predict the optimum performance of the MBT76, stochastic simulation iterations were performed with $\xi_1 = 8.0$ secs and $\xi_2 = 5.0$ secs. The results are shown in Table 21 with a derived 90% confidence interval

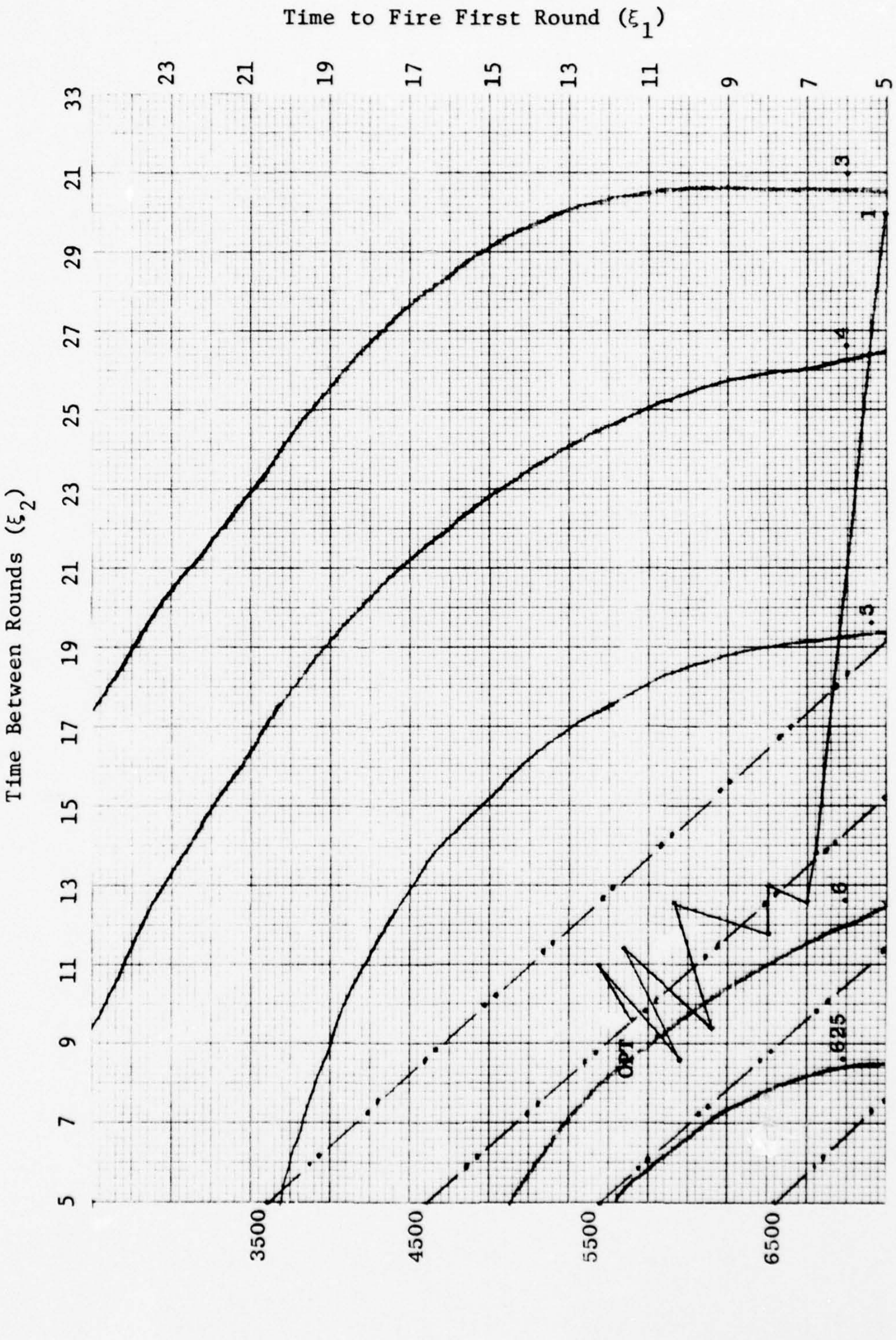

Figure 32. Convergence Movement of the Methodology.

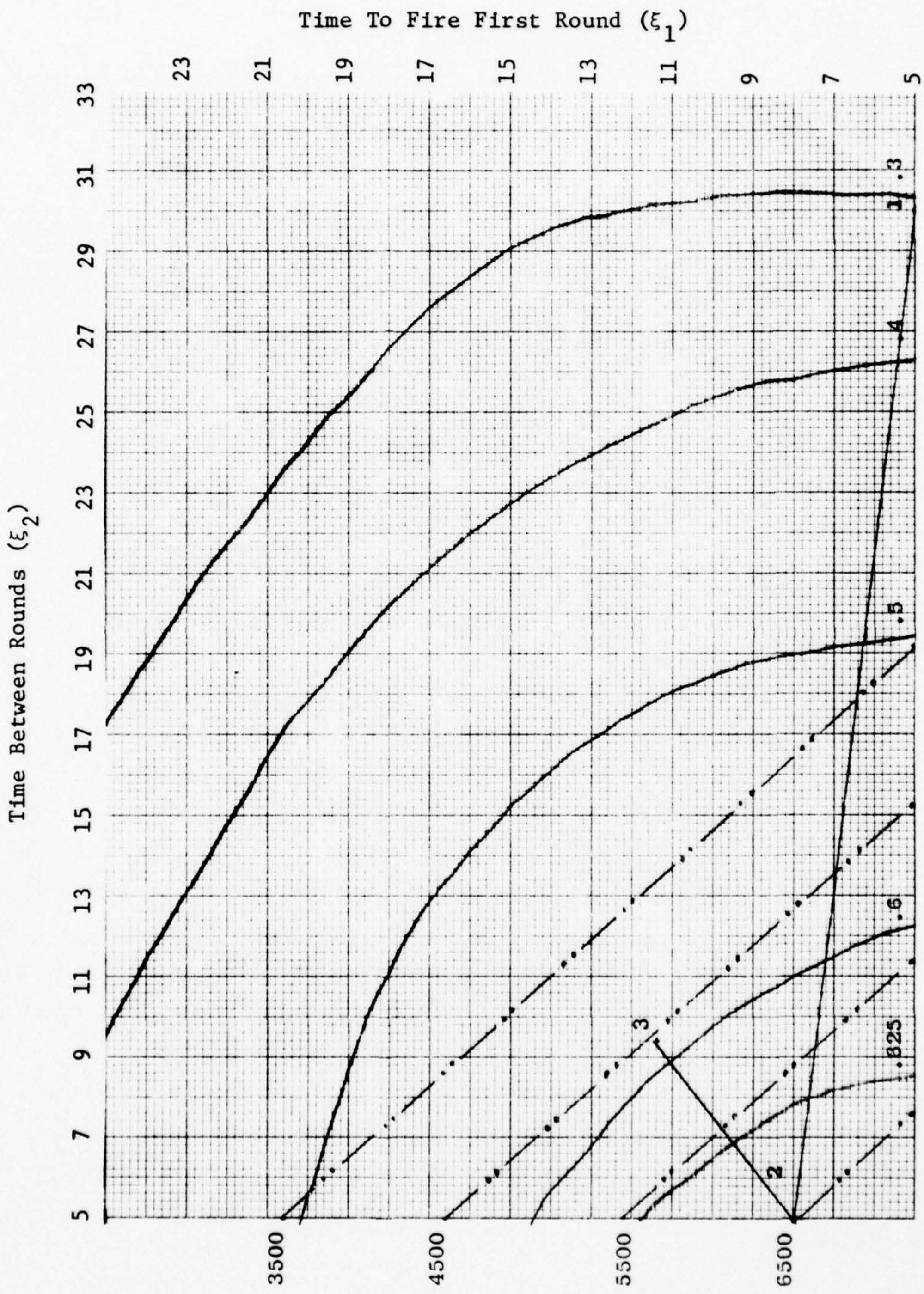

Figure 33. Validation of Methodology Convergence.

Table 20. AMSAA Tank Duel Model Output at $\xi_1 = 23.0$ and $\xi_2 = 18.0$.

.390	.406	.387	.407	.432
.372	.389	.420	.369	.345
.392	.387	.419	.382	

Table 21. AMSAA Tank Duel Model Output at $\xi_1 = 8.0$ and $\xi_2 = 5.0$.

.669	.691	.652	.665	.689
.678	.639	.674	.670	.689
.695	.720	.699	.690	

about the probability of victory of

$$.6435 \leq \mu y_1 \leq .7165 \tag{4.30}$$

From this analysis of training effects on MBT76 OT performance, it was apparent to the PM that his test personnel must receive further training. Indications were that when OT I data was simulated in two-sided combat, the MBT76 would not be victorious. Yet with proper crew training, the MBT76 would be victorious 68% of the time. Certainly furthur OT's must be conducted at a training level closer to optimum.

Much as a tactical unit commander would do, the PM and the AT designed a training program for the test personnel. Their objective was to maximize probability of victory. The test cycle timetable and budget, however, imposed constraints of no more than 50 hours dry training per crew, no more than 55 training rounds per crew, and no more than $5500.00 training cost per crew. With this problem formulation, the PM and AT began optimization utilizing the adapted Interactive Vector Maximal algorithm. Figure 34 graphs the four iterations of the methodology resulting in an optimum point of $\xi_1 = 10.7$ secs and $\xi_2 = 8.2$ secs. Output from the optimization methodology predicted that training to this proficiency would result in a probability of victory of .6099. The predicted training effort to arrive at this level was 41.9 hours of dry training per crew, 55.2 live rounds fired per crew, and a cost of $4982.62 per crew.

To confirm these results the AT ran the simulation at these levels yielding the results in Table 22 and a 90% confidence interval around the

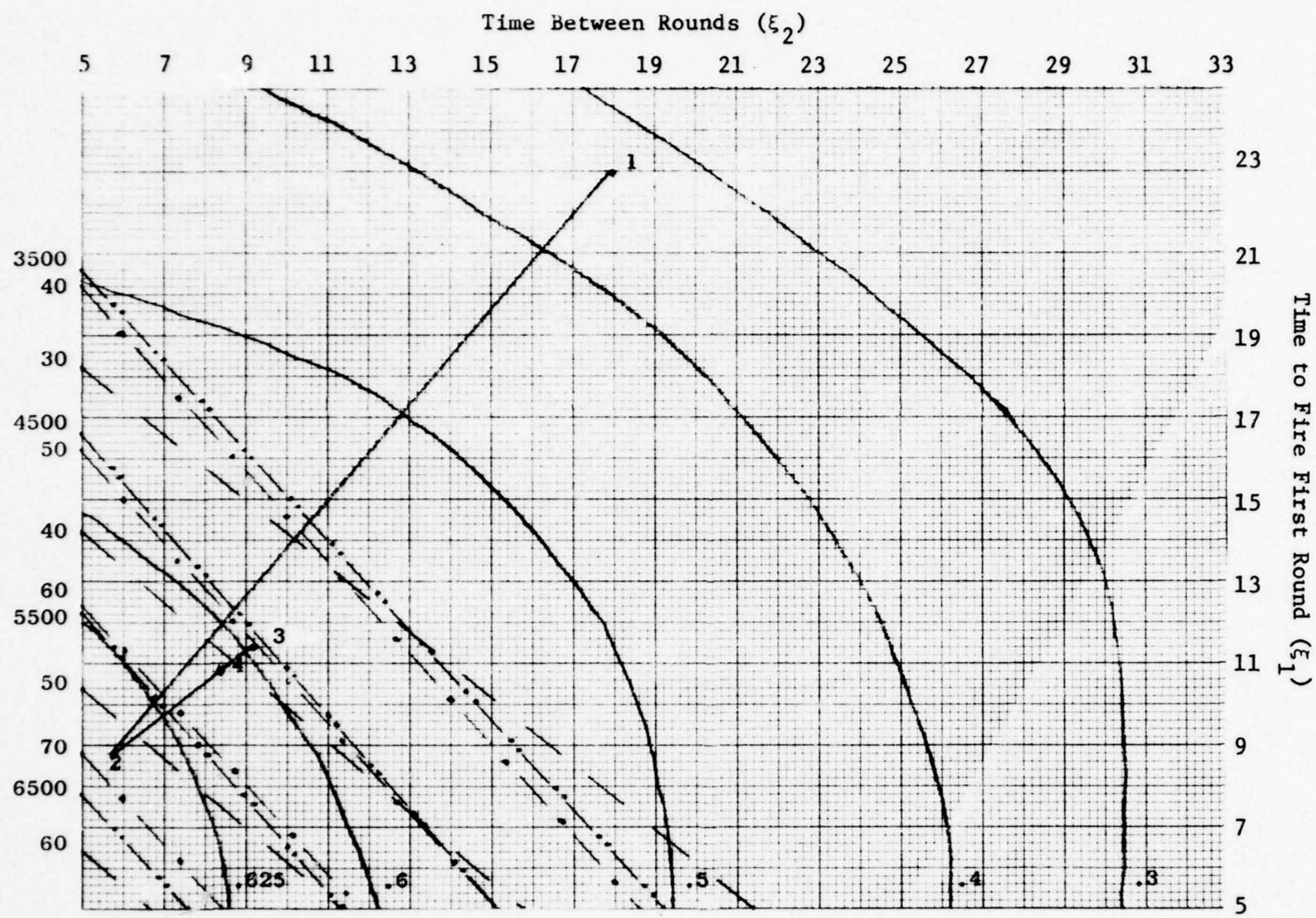

Figure 34. Movement of Optimization Methodology in Training Problem.

probability of victory of

$$.5377 \leq \mu y_1 \leq .6547 \quad (4.31)$$

Further sensitivity analysis around the optimum point was accomplished by iterating the adapted algorithm in varied uni-direction searches from the optimum point. The searches are listed in the following tables: Table 23 toward point (8.0,5.0), Table 24 toward point (8.0,15.0), Table 25 toward point (16.0, 5.0), and Table 26 toward point (16.0, 15.0).

Upon analyzing this sensitivity analysis, the PM was satisfied with the proposed training program and its crew performance objectives. Implimentation of the training program was begun immediately. Future OT reports to the ASARC included a section analyzing the training level of the test personnel and the effect of training on the performance of the MBT76 in two-sided, European type conflicts.

Table 22. AMSAA Tank Duel Model Output at $\xi_1 = 10.7$ and $\xi_1 = 8.2$.

.589	.592	.662	.619	.563
.567	.650	.596	.578	.590
.561	.587			

Table 23. Sensitivity Analysis Toward (8.0,5.0).

.6099	1.4548	41.9288	55.2458	4982.6208
.6129	1.4653	42.8618	56.3513	5082.1132
.6157	1.4757	43.7947	57.4567	5181.6056
.6185	1.4859	44.7276	58.5622	5281.0980
.6212	1.4959	45.6605	59.6677	5380.5904
.6239	1.5057	46.5934	60.7731	5480.0827
.6264	1.5154	47.5263	61.8786	5579.5751
.6289	1.5249	48.4593	62.9841	5679.0675
.6313	1.5341	49.3922	64.0896	5778.5599
.6336	1.5432	50.3251	65.1950	5878.0522
.6359	1.5522	51.2580	66.3005	5977.5446
.6381	1.5609	52.1909	67.4060	6077.0370
.6402	1.5694	53.1238	68.5114	6176.5294
.6422	1.5778	54.0568	69.6169	6276.0217
.6441	1.5860	54.9897	70.7224	6375.5141
.6460	1.5940	55.9226	71.8279	6475.0065

Table 24. Sensitivity Analysis Toward (8.0,15.0).

.6099	1.4548	41.9288	55.2458	4982.6208
.6077	1.4474	41.5250	54.5834	4923.0080
.6054	1.4395	41.1211	53.9211	4863.3951
.6030	1.4311	40.7173	53.2587	4803.7823
.6004	1.4221	40.3134	52.5963	4744.1694
.5977	1.4125	39.9096	51.9340	4684.5565
.5949	1.4024	39.5057	51.2716	4624.9437
.5920	1.3917	39.1019	50.6092	4565.3308
.5890	1.3805	38.6980	49.9469	4505.7180
.5859	1.3687	38.2941	49.2845	4446.1051
.5826	1.3563	37.8903	48.6221	4386.4922
.5792	1.3434	37.4864	47.9598	4326.8794
.5757	1.3299	37.0826	47.2974	4267.2665
.5721	1.3159	36.6787	46.6350	4207.6537
.5684	1.3013	36.2749	45.9727	4148.0408
.5645	1.2862	35.8710	45.3103	4088.4279
.5606	1.2705	35.4672	44.6479	4028.8151
.5565	1.2542	35.0633	43.9856	3969.2022
.5523	1.2374	34.6595	43.3232	3909.5894
.5480	1.2200	34.2556	42.6609	3849.9765

Table 25. Sensitivity Analysis Toward (16.0,5.0).

.6099	1.4548	41.9288	55.2458	4982.6208
.6096	1.4529	41.5893	55.0191	4962.2205
.6091	1.4505	41.2498	54.7924	4941.8202
.6083	1.4474	40.9102	54.5658	4921.4198
.6074	1.4437	40.5707	54.3391	4901.0195
.6064	1.4395	40.2312	54.1124	4880.6192
.6051	1.4346	39.8917	53.8858	4860.2188
.6036	1.4291	39.5521	53.6591	4839.8185
.6020	1.4230	39.2126	53.4324	4819.4182
.6002	1.4163	38.8731	53.2058	4799.0178
.5982	1.4090	38.5336	52.9791	4778.6175
.5960	1.4011	38.1940	52.7524	4758.2172
.5937	1.3926	37.8545	52.5257	4737.8168
.5911	1.3835	37.5150	52.2991	4717.4165
.5884	1.3737	37.1754	52.0724	4697.0162
.5855	1.3634	36.8359	51.8457	4676.6158
.5824	1.3525	36.4964	51.6191	4656.2155
.5791	1.3409	36.1569	51.3924	4635.8152
.5756	1.3288	35.8173	51.1657	4615.4148
.5720	1.3160	35.4778	50.9391	4595.0145

Table 26. Sensitivity Analysis Toward (16.0,15.0).

.6099	1.4548	41.9288	55.2458	4982.6208
.6051	1.4375	40.4489	53.4840	4824.0610
.6000	1.4198	38.9690	51.7222	4665.5012
.5948	1.4017	37.4891	49.9605	4506.9414
.5894	1.3831	36.0092	48.1987	4348.3816
.5838	1.3641	34.5294	46.4369	4189.8218
.5780	1.3446	33.0495	44.6751	4031.2620
.5721	1.3246	31.5696	42.9134	3872.7022
.5659	1.3042	30.0897	41.1516	3714.1424
.5596	1.2834	28.6098	39.3898	3555.5826
.5531	1.2621	27.1299	37.6280	3397.0228
.5463	1.2403	25.6500	35.8663	3238.4630
.5394	1.2181	24.1701	34.1045	3079.9032
.5324	1.1954	22.6902	32.3427	2921.3433
.5251	1.1723	21.2103	30.5809	2762.7835
.5177	1.1488	19.7304	28.8192	2604.2237
.5100	1.1248	18.2505	27.0574	2445.6639
.5022	1.1003	16.7706	25.2956	2287.1041
.4942	1.0754	15.2907	23.5338	2128.5443
.4860	1.0500	13.8108	21.7721	1969.9845

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The field of multiple response surface methodology was found to consist largely of applications of nonlinear programming techniques to problem formulations of a primary and a constraint response. Contemporary efforts continue to enhance this area with application of further nonlinear programming algorithms. This research is an initial effort to optimize multiple response surfaces by means of the Geoffrion-Dyer Interactive Vector Maximal algorithm.

A modified version of the Interactive Vector Maximal algorithm was found to be well suited to the optimization of multiple response surfaces. Various practical region of experimentation boundary definitions are easily incorporated into the methodology. Algorithm assumption violations were present in saddle and ridge systems. Methods for optimization in the presence of such assumption violations were devised. The methodology was shown to converge and to satisfy the Kuhn-Tucker conditions necessary for optimality. FORTRAN IV computer programs were written to perform the procedure on a CDC CYBER 74 computer.

It has been demonstrated that through computer simulation and response surface methodology, OTEA can extend the analysis, scope and optimization of OT results. A mutually supporting relationship between OT's and computer simulations was discussed. The importance of the

military decision maker and the benefits accrued by his participation in the methodology of this research have been discussed. An application of the methodology to the analysis of the effects of training in OT's has been demonstrated.

Recommendations

This research generated several recommendations. The suboptimization algorithm of the methodology should be investigated for an algorithm which would better optimize a saddle and/or a ridge system. A nonlinear algorithm such as Zoutendijk's Method or the Conjugate Direction Method should be considered. Another aid in this area might be the simultaneous utilization of a visual display of the response surface so that the decision maker might better follow the implications of his optimization movements. Some of the other multiple objective algorithms mentioned in Chapter II, such as SEMOPS, should be investigated for applicability to multiple response surface optimization. The design of OT's should be analyzed from a design of experiment viewpoint. Utilization of fractional designs would greatly reduce the number of replications, thereby perhaps making actual OT data available for analysis by this methodology. Finally OTEA should implement the methodology of this research to enhance and improve the resulting analysis of operational tests. There are several excellent military computer simulations available. Hopefully this research and its references can serve as a guide in the implementation of multiple response surface optimization and analysis.

APPENDIX A

This appendix contains two programs utilized to generate the normal deviates necessary for input to the AMSAA Tank Duel Model. The first program utilizes an internal CDC CYBER 74 U(0,1) generator to generate U(0,1) deviates. The second program transforms these uniform deviates to normal deviates of specified mean and variance through the use of the Fishman equations.

```
C
C*****THIS PROGRAM GENERATES U(0,1) DEVIATES AND STORES
C*****IN A FILE.
C
      PROGRAM UNGEN(INPUT,OUTPUT,TAPE3,TAPE5=INPUT,
     *  TAPE6=OUTPUT)
      DIMENSION RAN(200)
      NUM=200
      CALL RANSET(0)
      DO 200 I=1,NUM
      RAN(I)=RANF(0)
200   CONTINUE
      WRITE(3,*)(RAN(I),I=1,NUM)
      STOP
      END
```

```
C
C*****THIS PROGRAM TRANSFORMS U(0,1) DEVIATES INTO N(0,1)
C*****DEVIATES OF GIVEN MEAN AND VARIANCE.
C
      PROGRAM NORM(INPUT,OUTPUT,TAPE3,TAPE5=INPUT,
     *  TAPE6=OUTPUT)
      DIMENSION RAN(200),RANORM(200)
      NUM=200
      PI=2.*3.141592653
C
C*****THIS STATEMENT READS THE U(0,1) FROM A FILE.
C
      READ(3,*)(RAN(J),J=1,NUM)
C
C*****THIS SECTION COMPUTES THE NORMAL(0,1).
C
100   PRINT 548
548   FORMAT(*WHAT ARE NORMAL MEAN AND VARIANCE*)
      READ(5,*)ORMU,ORMVAR
C
C*****THESE ARE THE FISHMAN EQUATIONS.
C
      DO 550 J=1,NUM,2
      DUMMY=SQRT(-2.*ORMVAR*ALOG(RAN(J)))
      RANORM(J)=ORMU+DUMMY*COS(PI*RAN(J+1))
      RANORM(J+1)=ORMU+DUMMY*SIN(PI*RAN(J+1))
550   CONTINUE
      WRITE(6,*)ORMU,ORMVAR,(RANORM(J),J=1,NUM)
      GOTO 100
      STOP
      END
```

APPENDIX B

This appendix contains the AMSAA Tank Duel Model simulation modified for use in this research. Several of the inputs have been fixed or rendered stochastic as discussed in Chapter IV of this thesis. Following the listing of the simulation is an example of an input data file utilized by the simulation. Figure 26, page 78, is an example of the simulation's output.

```
C
C*****THIS IS THE AMSAA TANK DUEL SIMULATIØN MØDEL
C
      PRØGRAM TANK(INPUT,ØUTPUT,TAPE3,TAPE5=INPUT,TAPE6=ØUTPUT)
      DIMENSIØN TMDB(45),TMDR(45),SDB(45),SDR(45),SKB(45),SKR
      *(45)
      REAL KB(45),KR(45),M(40,40),N(40,40),NDF,NØDEC
      INTEGER RANGE
      DATA SIGMA,BLUE,RED,TCUT,LS,TFB,TTB,ID1,ID2,ID3,RANGE,
     1RELB,TFR,TTR,ID4,ID5,ID6,RELR/.5,4HBLUE,3HRED,120.0,0,
     2.86,7.,3HBLU,3HBLU,3HBLU,1000,.85,1.17,7.,3HRED,3HRED,
     33HRED,.825/
100   READ(3,912) TB1,TBS,BPH1,BPHH,BPHS,BPHL,BKH1,
     1BKHH,BKHS,BKHL,BS
      READ(3,912) TR1,TRS,RPH1,RPHH,RPHS,RPHL,RKH1,
     1RKHH,RKHS,RKHL,RS
      IF (LS.EQ.0) WRITE(6,904)BLUE,RED,TCUT
      IF (LS.NE.0) WRITE(6,905)BLUE,LS,RED,TCUT
      WRITE(6,916)RANGE,ID1,ID2,ID3,ID4,ID5,ID6
      WRITE(6,917)TFB,TTB,TB1,TBS,BPH1,BPHH,BPHS,BPHL,BKH1,BKHH,
     1BKHS,BKHL,BS,RELB,TFR,TTR,TR1,TRS,RPH1,RPHH,RPHS,RPHL,
     *RKH1
     2RKHH,RKHS,RKHL,RS,RELR
      CALL KASFT(KB,SKB,JØUTB,BKH1,BKHH,BKHS,BKHL,BPH1,BPHH,
     *BPHS,
     1BPHL,BS,RELB)
      CALL KASFT(KR,SKR,JØUTR,RKH1,RKHH,RKHS,RKHL,RPH1,RPHH,
     *RPHS,
     1RPHL,RS,RELR)
      JØUT=MIN0(40,MAX0(JØUTB,JØUTR))
      SFTB=0.0
      SFTR=0.0
      IF (TFB.GT.TFR) SFTB=TFB-TFR
      IF (TFR.GT.TFB) SFTR=TFR-TFB
      TMDB(1)=TB1
      SDB(1)=SIGMA
      DØ 120 I=2,45
120   CALL CØNLØG(TBS,SIGMA,TMDB(I-1),SDB(I-1),TMDB(I),SDB
     *(I))
      TMDR(1)=TR1
      SDR(1)=SIGMA
      IF (LS.EQ.0) GØTØ 130
C
C*****ADJUST RED TIMES FØR HEADSTART
C
      TSAVE=TMDR(1)
      CALL CØNLØG(TMDR(1),SDR(1),TMDB(LS),SDB(LS),TMDR(1),SDR
     *(1))
130   DØ 140 I=2,40
140   CALL CØNLØG(TRS,SIGMA,TMDR(I-1),SDR(I-1),TMDR(I),SDR(I))
```

```
      DØ 150 I=1,45
150   TMDB(I)=TMDB(I)+FLØAT(I-1)*TTB+SFTB
      IF (LS.LE.1) GØTØ 170
      TSAVE=TTB*(FLØAT(LS-1))
      DØ 160 I=1,40
160   TMDR(I)=TMDR(I)+TSAVE
170   DØ 180 I=1,40
180   TMDR(I)=TMDR(I)+FLØAT(I-1)*TTR+SFTR
C
C*****CØMPUTE AVERAGE NUMBER ØF RØUNDS FIRING ASSUMING NØ
C*****KILLS.
C
      RNDB=NDF(ALØG((TCUT/TMDB(1)))/SDB(1))
      RNDR=NDF(ALØG((TCUT/TMDR(1)))/SDR(1))
      L=40+LS
      DØ 190 I=2,L
190   RNDB=RNDB+NDF(ALØG((TCUT/TMDB(I)))/SDB(I))
      DØ 195 I=2,40
195   RNDR=RNDR+NDF(ALØG((TCUT/TMDR(I)))/SDR(I))
C
C*****M(I,J) GIVES THE PRØBABILITY THAT BLUE FIRES HIS
C*****I TH RØUND BEFØRE RED KILLS WITH HIS J TH, AND
C*****BØTH BEFØRE TCUT.
C*****N(I,J) GIVES THE RESULTS FØR RED.
C
      DØ 200 I=1,JØUT
      DØ 200 J=1,JØUT
      M(I,J)=PABAT(TCUT,TMDB(I+LS),TMDR(J),SDB(I+LS),SDR(J))
200   N(I,J)=PABAT(TCUT,TMDR(I),TMDB(J+LS),SDR(I),SDB(J+LS))
      DØ 210 I=1,JØUT
      DØ 210 J=2,JØUT
      K=JØUT+2-J
      M(I,K)=M(I,K)-M(I,K-1)
210   N(I,K)=N(I,K)-N(I,K-1)
      PWINB=0.0
      PWINR=0.0
      ANRB=0.0
      ANRR=0.0
      IF (LS.EQ.0) GØTØ 225
      DØ 220 I=1,LS
      TSAVE=NDF(ALØG((TCUT/TMDB(I)))/SDB(I))
      PWINB=PWINB+KB(I)*TSAVE
220   ANRB=ANRB+FLØAT(I)*KB(I)*TSAVE
      ANRB=ANRB+FLØAT(LS)*SKB(LS)*KR(1)*N(1,1)
225   DØ 230 LQQ=1,JØUT
      PWINB=PWINB+M(LQQ,1)*KB(LS+LQQ)
230   ANRB=ANRB+FLØAT(LS+LQQ)*M(LQQ,1)*(KB(LS+LQQ)+KR(1)*SKB(
     1LS+LQQ))
      DØ 235 LQ=2,JØUT
      DØ 235 LQQ=1,JØUT
      PWINB=PWINB+KB(LS+LQQ)*M(LQQ,LQ)*SKR(LQ-1)
```

```
235   ANRB=ANRB+FLØAT(LS+LQQ)*M(LQQ,LQ)*(KB(LS+LQQ)+KR(LQ)*
     1SKB(LS+LQQ))*SKR(LQ-1)
      IF (LS.EQ.0) GØTØ 245
      DØ 240 LQQ=1,JØUT
      PWINR=PWINR+KR(LQQ)*N(LQQ,1)*SKB(LS)
240   ANRR=ANRR+FLØAT(LQQ)*N(LQQ,1)*(KR(LQQ)+SKR(LQQ)*KB(LS+1))
     1*SKB(LS)
      GØTØ 260
245   DØ 250 LQQ=1,JØUT
      PWINR=PWINR+KR(LQQ)*N(LQQ,1)
250   ANRR=ANRR+FLØAT(LQQ)*N(LQQ,1)*(KR(LQQ)+SKR(LQQ)*KB(1))
260   DØ 270 LQ=2,JØUT
      DØ 270 LQQ=1,JØUT
      PWINR=PWINR+KR(LQQ)*N(LQQ,LQ)*SKB(LQ+LS-1)
270   ANRR=ANRR+FLØAT(LQQ)*N(LQQ,LQ)*SKB(LQ+LS-1)*(KR(LQQ)+
     1SKR(LQQ)*KB(LS+LQ))
      NØDEC=1.0-PWINB-PWINR
      ANRB=ANRB+NØDEC*RNDB
      ANRR=ANRR+NØDEC*RNDR
      WRITE(6,915)BLUE,PWINB,RED,PWINR,NØDEC,BLUE,ANRB,RED,
     1ANRR
      GØTØ 100
904   FØRMAT(///10X,*A MEETING ENGAGEMENT BETWEEN*,A10,*AND*
     1,A10/10X,*THE TIME LIMIT IS*,F8.2,*SECØNDS*)
905   FØRMAT(///10X,A10,* THE DEFENDER HAS A*,I2,*RØUND
     &HEADSTART*,
     1 /10X,A10,* IS THE ATTACKER*/10X,*THE TIME LIMIT IS*,
     2 F8.2,* SECØNDS*)
912   FØRMAT(2F5.2,9F5.4)
915   FØRMAT(10X,*PRØB(*,A5,* WINS)=*,F6.3/10X,*PRØB(*,A5,
     1* WINS)=*,F6.3/10X,*PRØB(NØ DECISIØN)=*,F6.3/10X,*E(RDS
     &FØR*,
     2 A5,2H)=,F9.3/10X,*E(RDS FØR *,A5,2H)=,F9.3)
916   FØRMAT(10X,*RANGE IS*,I5,* METERS*/10X,*BLUE DATA IS *
     1,3(A3,1X)/10X,*RED DATA IS *,3(A3,1X)/9X,3HTFL,4X,2HTT,
     24X,2HT1,4X,2HTS,2X,3HPH1,2X,3HPHH,2X,3HPHS,2X,3HPHL,2X,
     33HKH1,2X,3HKHH,2X,3HKHS,2X,3HKHL,3X,2HPS,3X,3HREL)
917   FØRMAT(1X,4HBLUE,1X,4F6.2,9F5.3,F5.2/1X,3HRED,2X,4F6.2,
     19F5.3,F5.2)
      STØP
      END
C
C*****THIS SUBRØUTINE CØMPUTES THE MEDIAN AND STANDARD
C*****DEVIATIØN FØR CØNVØLUTIØN.
C
      SUBRØUTINE CØNLØG(XI,SIGX,ETA,SIGY,ZETA,SIGZ)
      XBAR=XI*EXP(.5*SIGX*SIGX)
      YBAR=ETA*EXP(.5*SIGY*SIGY)
      SSX=XBAR*XBAR*(EXP(SIGX*SIGX)-1.0)
      SSY=YBAR*YBAR*(EXP(SIGY*SIGY)-1.0)
```

```
      ESIGZ=1.0+((SSX+SSY)/((XBAR+YBAR)**2))
      ZETA=(XBAR+YBAR)/SQRT(ESIGZ)
      SIGZ=SQRT(ALØG(ESIGZ))
      RETURN
      END
C
C*****THIS SUBRØUTINE CØMPUTES THE KILL AND SURVIVAL
C*****FØR THE TWØ TANKS.
C
      SUBRØUTINE KASFT(K,SK,JØUT,KH1,KHH,KHS,KHL,PH1,PHH,PHS,
     1PHL,S,R)
      DIMENSIØN SK(45)
      REAL K(45),KH1,KHH,KHS,KHL,L
      DØ 100 I=2,45
      K(I)=0.0
100   SK(I)=0.0
      K(1)=PH1*KH1*R
      SK(1)=1.0-K(1)
      L=1.0-S
      X2=PH1*(1.0-KH1*R)
      X3=(1.0-PH1)*S
      X4=(1.0-PH1)*L
      A12=PHH*KHH*R
      A13=PHS*KHS*R
      A14=PHL*KHL*R
      A22=PHH*(1.0-KHH*R)
      A23=PHS*(1.0-KHS*R)
      A24=PHL*(1.0-KHL*R)
      A32=(1.0-PHH)*S
      A33=(1.0-PHS)*S
      A34=(1.0-PHL)*S
      A42=(1.0-PHH)*L
      A43=(1.0-PHS)*L
      A44=(1.0-PHL)*L
      DØ 130 I=2,45
      K(I)=A12*X2+A13*X3+A14*X4
      X3P=A32*X2+A33*X3+A34*X4
      X2P=A22*X2+A23*X3+A24*X4
      X4P=A42*X2+A43*X3+A44*X4
      X2=X2P
      X3=X3P
      X4=X4P
      JØUT=I
      SK(I)=SK(I-1)-K(I)
      IF (I.LT.11) GØTØ 130
      IF (SK(I-5).LT..0005) GØTØ 135
130   CØNTINUE
135   RETURN
      END
C
C*****THIS FUNCTIØN CØMPUTES THE ELEMENTS ØF
```

```
C*****M(I,J) AND N(I,J).
C
      FUNCTIØN PABAT(T,TA,TB,SA,SB)
      REAL NDF
      EXTERNAL PAFINT
      CØMMØN/PAF/A,B
      IF (SA.GE.0.) GØTØ 2
      X=T
      A=TA
      B=TB
      GØTØ 7
2     X=T/TA
      IF (X.GT..0000001) GØTØ 5
      PABAT=0.
      RETURN
5     X=ALØG(X)/SA
      A=ALØG(TA/TB)/SB
      B=SA/SB
7     C=B*B+1.
      D=A/SQRT(C)
      E=A+B*X
      IF (X*X+E*E.LT.25.) GØTØ 30
      IF (E.LT.0.) GØTØ 10
      PABAT=1.-NDF(D)
      RETURN
10    IF (X.GT.0.) GØTØ 20
      PABAT=NDF(X)
      RETURN
20    PABAT=NDF(X)
      IF(A*B/C.LT.X) RETURN
      PABAT=PABAT-NDF(D)
      RETURN
30    F=SQRT(25.*C-A*A)
      AB=A*B
      UZ=-A/B
      UIM=(-AB-F)/C
      UIP=(-AB+F)/C
      BR=-5.
      IF (UZ.GE.-5.) BR=UIM
      TS=5.
      IF (UZ.LT.5.) TS=UIP
      IF (X-BR.LE.TS-X) GØTØ 40
      CALL SAMSØN(PAFINT,G,X,TS,.0001)
      PABAT=1.-NDF(D)-NDF(TS)+NDF(X)+G
      PABAT=ABS(PABAT)
      RETURN
40    CALL SAMSØN(PAFINT,G,BR,X,.0001)
      PABAT=NDF(X)-G
      PABAT=ABS(PABAT)
      RETURN
```

```
      END
      FUNCTIØN PAFINT(U)
      REAL NDF
      CØMMØN/PAF/A,B
      PAFINT=.3989422803*EXP(-U*U/2.)*NDF(A+B*U)
      RETURN
      END
      SUBRØUTINE SAMSØN(FUN,R,A,B,EPS)
      IF (B-A.GE..0001) GØTØ 18
      R=0.
      RETURN
18    EPSI=EPS
      NT=0
      N=1
      M=1
      XU=B
      XL=A
      H=(XU-XL)/2.
      HBAR=0.
      FJ=H*(FUN(XU)+FUN(XL))
      FIBAR=10000.
1     S=0.
      X=XL+H
2     S=S+FUN(X)
      X=X+HBAR
      M=M-1
      IF (M) 3,3,2
3     FI=FJ+4.*H*S
      IF (FIBAR) 4,5,4
4     ERR=ABS((FIBAR-FI)/FIBAR)
      IF (ERR-EPSI) 9,5,5
5     IF (NT-13) 7,9,9
7     NT=NT+1
      FIBAR=FI
      FJ=(FI+FJ)/4.
      HBAR=H
      H=H/2.
      N=2*N
      M=N
      GØTØ 1
9     R=FI/3.
      RETURN
      END
```

```
C
C*****THIS FUNCTIØN IS THE CU MULATIVE NØRMAL DISTRIBUTIØN
C
      REAL FUNCTIØN NDF(X)
      NDF=0.
      AX=ABS(X)
      IF (AX.GE.5.) GØTØ 3
      NDF=(((((.5383E-5*AX+.488906E-4)*AX+.380036E-4)*AX
     1+.0032776263)*AX+.0211410061)*AX+.0498673469)*AX+1.
      NDF=.5/((NDF**8)**2)
3     IF (X.GE.0.) NDF=1.-NDF
      RETURN
      END
```

```
23.0018.00.7997.8394.7932.8023.5417.8509.4977.4936.6000
08.5410.04.6754.7288.7192.6658.3920.8280.4780.4700.5398
23.0018.00.7340.8432.8220.8268.5016.8405.5029.5119.6000
08.0411.76.5681.7692.7328.5529.4113.8200.4135.5238.4964
23.0018.00.7398.8720.8417.7770.4818.8761.5435.4491.6000
09.7609.98.6290.7828.6420.6599.4455.7613.4477.4436.4980
23.0018.00.7831.8917.8016.7523.5500.8389.4797.4947.6000
07.9710.98.6494.6920.7280.6499.4529.7635.4619.4935.5795
23.0018.00.8128.8516.7818.8372.4788.8600.4633.5860.6000
08.9811.32.5125.7780.7200.5769.3991.8738.4297.4447.5217
23.0018.00.7525.6318.8500.7487.5192.8671.5226.4913.6000
09.3209.04.6423.7700.6613.5796.4133.7955.5360.4726.5233
23.0018.00.7225.9000.7788.7989.5328.8197.4550.4734.6000
07.0411.20.6302.7113.6635.7167.4413.7977.4050.4234.5203
23.0018.00.8254.8268.8192.8158.4420.8646.5113.4902.6000
09.2011.00.5417.7135.7738.6194.4613.7936.4402.4188.5271
23.0018.00.7180.8692.8328.7029.5280.8605.4688.5238.6000
09.0009.35.5450.8238.6955.6221.4738.8029.4658.4427.5338
23.0018.00.7790.8828.7420.8099.5200.8298.5158.4927.6000
07.5309.58.7112.7455.6977.6171.4585.8119.4827.4343.5571
23.0018.00.7994.7920.8280.7999.4613.8309.5085.5327.6000
07.5812.35.5933.7477.6936.6286.4536.8434.4198.4605.4874
23.0018.00.6625.8780.8200.7269.4635.8885.4843.5036.6000
10.3510.39.5965.7436.7029.6398.4539.7491.4913.4295.5100
23.0018.00.7923.8700.7613.7296.5738.8477.4698.5105.6000
08.3910.44.5904.7529.7119.6791.4368.7797.4077.4513.5211
23.0018.00.7802.8113.7635.8667.4955.8488.5039.5413.6000
08.4410.34.6043.7619.7435.5618.4533.7947.4630.4615.4979
```

APPENDIX C

This appendix contains two procedure files and their respective input data files for utilization with the SPSS Multiple Linear Regression program. The first procedure and input files are examples of those utilized for multiple linear regression. The second files are examples of those utilized for multiple polynomial regression. Examples of SPSS Multiple Linear Regression output can be found throughout Chapter IV of this thesis.

```
 5.     RUN NAME
 5.005  MULTIPLE LINEAR REGRESSIØN ØN TANK
10.     VARIABLE LIST
10.005  TB1,TBS,PS,PV,ER
30.     INPUT FØRMAT
30.005  FIXED(3F6.3,F4.3,F5.3)
40.     NØ. ØF CASES
40.005  14
50.     REGRESSIØN
50.001  VARIABLES=TB1,TBS,PS,PV,ER/
50.002  REGRESSIØN=PV WITH TB1,TBS,PS(2)/
50.003  REGRESSIØN=ER WITH TB1,TBS,PS(4)
```

```
-1.000-1.000-1.000.4070.795
01.000-1.000-1.000.3410.738
-1.00001.000-1.000.3470.709
01.00001.000-1.000.3070.581
-1.000-1.00001.000.4500.931
01.000-1.00001.000.3040.612
-1.00001.00001.000.3560.721
01.00001.00001.000.3100.637
00.00000.00000.000.3180.629
00.00000.00000.000.3010.576
00.00000.00000.000.3420.729
00.00000.00000.000.3290.739
00.00000.00000.000.3710.690
00.00000.00000.000.3360.673
```

```
 5.     RUN NAME
 5.005  MULTIPLE LINEAR REGRESSIØN ØN TANK
10.     VARIABLE LIST
10.005  TB1,TBS,PV,ER
30.     INPUT FØRMAT
30.005  FIXED(2F6.3,F4.3,F5.3)
40.     NØ. ØF CASES
40.005  13
41.0    CØMPUTE
41.005  TB12=TB1*TB1
44.0    CØMPUTE
44.005  TB1TBS=TB1*TBS
46.0    CØMPUTE
46.005  TBS2=TBS*TBS
50.     REGRESSIØN
50.001  VARIABLES=TB12,TB1TBS,TBS2,TB1
50.002  ,TBS,PV,ER/
50.003  REGRESSIØN=PV WITH TB12,TB1TBS
50.004  ,TBS2,TB1,TBS(2)/
50.005  REGRESSIØN=ER WITH TB12,TB1TBS
50.006  ,TBS2,TB1,TBS(4)
```

```
-1.000-1.000.6691.635
01.000-1.000.5811.315
-1.00001.000.5381.235
01.00001.000.4601.021
00.00000.000.5771.337
00.00000.000.5851.380
00.00000.000.5811.366
00.00000.000.5731.332
00.00000.000.6091.426
-1.41400.000.5911.404
01.41400.000.5181.148
00.000-1.414.6171.504
00.00001.414.5331.092
```

APPENDIX D

This appendix contains the programs necessary for the adapted Interactive Vector Maximal algorithm. The first program is an interactive data program which queries the decision maker for necessary data and stores that data in a data file. Figure 18, page 60, is an example of the output from and the input to this program. The program allows a maximum of 10 response equations and 5 independent variables. The coefficients of the response equations are input in the following order:

$$x_i^2 \quad i = 1,\ldots,5,$$

$$x_{1i} \quad i = 2,3,4,5,$$

$$x_{2i} \quad i = 3,4,5,$$

$$x_{3i} \quad i = 4,5,$$

$$x_{45},$$

$$x_i \quad i = 1,\ldots,5$$

and the constant term. The gradient coefficients are input as $x_i, i = 1,\ldots,5$, and the constant term. The region of interest boundaries are the limits on the region of experimentation utilized in the second order design for the primary or all response functions. The limits must coincide to prevent extrapolation of an equation outside its region of experimentation. During optimization, these limits will not be exceeded, thus preventing extrapolation. The second program is an interactive program which utilizes input from both the first program of this appendix and

from the decision maker to perform iterations of the adapted Interactive Vector Maximal algorithm. Figure 19, page 61, is an example of the output from and the input to this second program.

Within the program, ZX3LP is called as a subroutine. This subroutine is part of the IMSL library available on the Georgia Tech CDC CYBER 74. The library subroutine ZX3LP accepts input for a linear programming optimization problem and utilizes the simplex method to optimize the problem. Also utilized in conjunction with the second program of this appendix is the Bazaraa Cyclic Coordinate Algorithm for Optimizing Penalty Functions computer program (5) available in the Georgia Tech ISyE computer library. If the boundary definitions of the suboptimization problem are nonlinear, the second program of this appendix terminates after outputing the objective function coefficients of the suboptimization problem. The Bazaraa program is then utilized to compute the optimum search direction. This new direction is then input back into the main program.

```
C
C*****THIS PRØGRAM INPUTS DATA INTØ A DATA FILE FØR
C*****ØPTIMIZATIØN BY THE INTERACTIVE VECTØR MAXIMAL
C*****ALGØRITHM.
C
      PRØGRAM DATAPRØ(INPUT,ØUTPUT,TAPE3,TAPE5=INPUT,TAPE6=)
     *ØUTPUT)
      DIMENSIØN X(5),REQ(10,21),NAME(10),REQG(50,6),BØUN(5,2),
     1SA(50,5),SB(50)
      WRITE (6,100)
100   FØRMAT (*INPUT NUMBER ØF RESPØNSE EQUATIØNS*)
      READ (5,*)NREQ
      WRITE (3,*)NREQ
      WRITE (6,101)
101   FØRMAT (*INPUT NUMBER ØF INDEPENDENT VARIABLES (X"S)*)
      READ (5,*)NX
      WRITE (3,*)NX
      WRITE (6,102)
102   FØRMAT (*INPUT INITIAL VALUE ØF INDEPENDENT VARIABLES
     *WITH . AND
     1,*)
      READ (5,*)(X(IX),IX=1,NX)
      WRITE (3,*)(X(IX),IX=1,NX)
      DØ 301 IM=1,NREQ
      WRITE (6,103) IM
103   FØRMAT (*INPUT CØEFFICIENTS ØF RESPØNSE EQUATIØN*,I2)
      READ (5,*)(REQ(IM,IC),IC=1,21)
      WRITE (3,*)(REQ(IM,IC),IC=1,21)
301   CØNTINUE
      WRITE (6,107)
107   FØRMAT (*INPUT RESPØNSE EQUATIØN NAMES IN GRØUPS ØF TEN
     *LETTERS*/
     1*AND SPACES, RIGHT JUSTIFIED, ØNE PER LINE*)
      READ (5,108)(NAME(IN),IN=1,NREQ)
      WRITE (3,108)(NAME(IN),IN=1,NREQ)
108   FØRMAT (A10)
      DØ 312 IF=1,NREQ
      DØ 313 JX=1,5
      WRITE (6,116) IF,JX
116   FØRMAT (*INPUT CØEFFICIENTS ØF GRADIENT F*,I2,*X*,I2)
      READ (5,*)(REQG(JC,KC),KC=1,6)
      WRITE (3,*)(REQG(JC,KC),KC=1,6)
      JC=JC+1
313   CØNTINUE
312   CØNTINUE
      WRITE (6,114)
114   FØRMAT (*INPUT REGIØN ØF INTEREST BØUNDARY DEFINITIØN, I
     *FØR*/
     1*INTEGER, L FØR LINEAR, ØR N FØR NØNLINEAR*)
      READ (5,115) NBØN
```

```
      WRITE (3,115)NBØN
115   FØRMAT (A1)
      IF (NBØN.EQ.1HI) GØTØ 210
      IF (NBØN.EQ.1HL) GØTØ 231
      IF (NBØN.EQ.1HN) GØTØ 232
210   DØ 319 KB=1,NX
      WRITE (6,117)KB
117   FØRMAT (*INPUT LØWER AND UPPER BØUNDS ØF X*,I1)
      READ(5,*)(BØUN(KB,LB),LB=1,2)
      WRITE (3,*)(BØUN(KB,LB),LB=1,2)
319   CØNTINUE
      GØTØ 232
231   WRITE (6,135)
135   FØRMAT (*INPUT NUMBER ØF LESS THAN ØR EQUAL CØNSTRAINTS*)
      READ (5,*)M1
      WRITE (3,*)M1
      WRITE (6,136)
136   FØRMAT (*INPUT NUMBER ØF EQUALITY CØNSTRAINTS*)
      READ (5,*)M2
      WRITE (3,*)M2
      IAS=M1+M2+2
      WRITE (3,*)IAS
      IF (M1.EQ.0) GØTØ 330
      DØ 330 IM1=1,M1
      WRITE (6,137)NX,IM1
137   FØRMAT (*INPUT *,I1,* CØEFFICIENTS ØF LESS THAN
     *CØNSTRAINT*,I2)
      READ (5,*)(SA(IM1,JM1),JM1=1,NX)
      WRITE (3,*)(SA(IM1,JM1),JM1=1,NX)
330   CØNTINUE
      IF (M2.EQ.0) GØTØ 331
      DØ 331 IM2=1,M2
      WRITE (6,138)NX,IM2
138   FØRMAT (*INPUT *,I1,* CØEFFICIENTS ØF EQUALITY CØNSTRAINT
     **,I2)
      READ (5,*)(SA((M1+IM2),JM2),JM2=1,NX)
      WRITE (3,*)(SA((M1+IM2),JM2),JM2=1,NX)
331   CØNTINUE
      ISB=M1+M2
      WRITE (6,139)
139   FØRMAT (*INPUT RHS ØF CØNSTRAINTS AS INPUT ABØVE*)
      READ (5,*)(SB(JSB),JSB=1,ISB)
      WRITE (3,*)(SB(JSB),JSB=1,ISB)
      GØTØ 232
232   ENDFILE 3
      REWIND 3
      STØP
      END
```

```
C----
C*****ADAPTED INTERACTIVE VECTØR MAXIMAL ØPTIMIZATIØN
C*****ALGØRITHM.
C
       PRØGRAM ØPTIMIZ(INPUT,ØUTPUT,TAPE3,TAPE5=INPUT,TAPE6=
      *ØUTPUT)
       DIMENSIØN SF(10),SY(10),W(10),DF(10,20),BB(10),NAME(10),
      *G(20),
      1REQG(50,6),REQJ(10,5),WG(5),BØUN(5,2),D(5),F(10),REQ(10,
      *21),
      1X(5),Y(5),Z(5),SA(50,5),SB(50),PSØL(5),DSØL(50),RW(2650)
      *,IW(172)
       DØ 305 II=1,5
       X(II)=0.
305    CØNTINUE
       W(1)=1.
C
C*****THIS SECTIØN READS INPUT DATA FRØM A DATA FILE.
C
       READ (3,*)NREQ
       READ (3,*)NX
       READ (3,*)(X(IX),IX=1,NX)
       DØ 301 IM=1,NREQ
       READ (3,*)(REQ(IM,IC),IC=1,21)
301    CØNTINUE
       READ (3,1081)(NAME(IN),IN=1,NREQ)
1081   FØRMAT (A10)
       JC=1
       DØ 312 IF=1,NREQ
       DØ 313 JX=1,5
       READ (3,*)(REQG(JC,KC),KC=1,6)
       JC=JC+1
313    CØNTINUE
312    CØNTINUE
       READ (3,1151)NBØN
1151   FØRMAT (A1)
       IF (NBØN.EQ.1HL) GØTØ 233
       IF (NBØN.EQ.1HI) GØTØ 234
       IF (NBØN.EQ.1HN) GØTØ 215
234    DØ 319 KB=1,NX
       READ (3,*)(BØUN(KB,LB),LB=1,2)
319    CØNTINUE
       GØTØ 215
C
C*****THIS SECTIØN PRESENTS THE DECISIØN MAKER WITH
      *ALTERNATIVES
C*****AND READS HIS TRADEØFF INPUTS.
C
233    READ (3,*)M1
       READ (3,*)M2
       READ (3,*)IAS
```

```
      ISB=M1+M2
      DØ 332 ISA=1,ISB
      READ (3,*)(SA(ISA,JSA),JSA=1,NX)
332   CØNTINUE
      READ (3,*)(SB(JSB),JSB=1,ISB)
      GØTØ 215
215   CALL REQEV(NREQ,F,NX,REQ,X)
      DØ 324 MS=1,NREQ
      SF(MS)=F(MS)
324   CØNTINUE
      JC=1
      LC=1
      L=1
      WRITE (6,104)
104   FØRMAT (*INPUT PERTURBATIØN ØF F(1), IN FAVØRABLE
     *DIRECTIØN*)
      READ (5,*)DFØNE
      BB(1)=F(1)+DFØNE
      DØ 308 JB=2,NREQ
      BB(JB)=F(JB)
308   CØNTINUE
      DØ 307 KT=2,NREQ
      WRITE (6,105)KT
105   FØRMAT (*INPUT PERTURBATIØN ØF F(*,I2,*), IN FAVØRABLE
     *DIRECTIØN*)
      READ (5,*)DF(KT,L)
204   IF (KT.EQ.2) GØTØ 200
      BB(KT-1)=F(KT-1)
200   BB(KT)=F(KT)-DF(KT,L)
      WRITE (6,106)
106   FØRMAT (25X,1HA,16X,1HB)
      DØ 309 NW=1,NREQ
      WRITE (6,109)NAME(NW),F(NW),BB(NW)
109   FØRMAT (A10,10X,F10.5,5X,F10.5)
309   CØNTINUE
      WRITE (6,110)
C
C*****THIS SECTIØN ADJUSTS THE ALTERNATIVES PRESENTED TØ
C*****THE DECISIØN MAKER UNTIL HE IS INDIFFERENT.
C
110   FØRMAT (*WHICH DØ YØU PREFER.  IF YØU ARE INDIFFERENT
     *TYPE I.*)
      READ (5,111)NDEC
111   FØRMAT (A1)
      IF (NDEC.EQ.1HI) GØTØ 201
      IF (NDEC.EQ.1HA) GØTØ 202
      DF(KT,L+1)=2*DF(KT,L)
      L=L+1
      GØTØ 204
206   WRITE (6,106)
```

```
      BB(KT)=F(KT)-DF(KT,L)
      DØ 310 JW=1,NREQ
      WRITE (6,109)NAME(JW),F(JW),BB(JW)
310   CØNTINUE
      WRITE (6,110)
      READ (5,111)NDEC
      IF (NDEC.EQ.1HA) GØTØ 203
      IF (NDEC.EQ.1HB) GØTØ 208
      GØTØ 201
202   G(L)=DF(KT,L)
203   DF(KT,L+1)=DF(KT,L)-(G(L)/2.)
      G(L+1)=G(L)/2.
      L=L+1
      GØTØ 206
208   DF(KT,L+1)=DF(KT,L)+(G(L)/2.)
      G(L+1)=G(L)/2.
      L=L+1
      GØTØ 206
C
C*****THIS SECTIØN CØMPUTES THE TRADEØFF VALUES.
C
201   W(KT)=(DFØNE)/(DF(KT,L))
307   CØNTINUE
      WRITE (6,112)
112   FØRMAT (*THE TRADEØFFS ARE*)
      DØ 311 LT=1,NREQ
      WRITE (6,113)NAME(LT),W(LT)
113   FØRMAT (A10,10X,F10.5)
311   CØNTINUE
C
C*****THIS SECTIØN CØMPUTES THE CØEFFICIENTS ØF THE
C*****SUBØPTIMIZATIØN ØBJECTIVE FUNCTIØN.
C
      DØ 314 IJ=1,NREQ
      DØ 315 JJ=1,5
      E=0.
      DØ 316 JS=1,NX
      E=E+(REQG(LC,JS))*X(JS)
316   CØNTINUE
      REQJ(IJ,JJ)=E+(REQG(LC,6))
      LC=LC+1
315   CØNTINUE
314   CØNTINUE
      DØ 317 KW=1,NX
      WG(KW)=0.
      DØ 318 LW=1,NREQ
      WG(KW)=WG(KW)+(W(LW)*REQJ(LW,KW))
318   CØNTINUE
      WRITE (6,*)WG(KW)
317   CØNTINUE
```

```
      DØ 1 IIW=1,NX
      WRITE (6,*) WG(IIW)
1     CØNTINUE
C
C*****THIS SECTIØN PERFØRMS THE SUBØPTIMIZATIØN.
C
      IF (NBØN.EQ.1HI) CALL SINT(Y,WG,BØUN,NX)
      IF (NBØN.EQ.1HL) CALL ZX3LP(SA,50,SB,WG,NX,M1,M2,S,Y
     1DSØL,RW,IW,IER)
      IF (NBØN.EQ.1HN) CALL NLP(Y,NX), RETURNS(214,999)
214   DØ 321 ID=1,NX
      D(ID)=Y(ID)-X(ID)
321   CØNTINUE
      WRITE (6,118)
118   FØRMAT (*NEW DECISIØN VECTØR*)
      DØ 322 JD=1,NX
      WRITE (6,119)JD,Y(JD)
119   FØRMAT (*Y*,I1,5X,F10.5)
322   CØNTINUE
      WRITE (6,120)
120   FØRMAT (*NEW ØPERATING PØINT*)
      CALL REQEV(NREQ,F,NX,REQ,Y)
      DØ 323 IY=1,NREQ
      WRITE (6,121) F(IY)
121   FØRMAT (F10.5)
323   CØNTINUE
      WRITE (6,122)
C
C*****THIS SECTIØN PERFØRMS THE STEP-SIZE ØPTIMIZATIØN.
C
122   FØRMAT (*INPUT NUMBER ØF PØINTS TØ SEE IN STEP SIZE*)
      READ (5,*)KS
      T=1./(KS-1)
      DØ 325 NS=1,NREQ
      SY(NS)=F(NS)
325   CØNTINUE
      WRITE (6,123)(SF(MW),MW=1,NREQ)
123   FØRMAT (5F12.4/5X,5F12.4)
      KZ=KS-2
      DØ 326 MT=1,KZ
      DØ 327 MX=1,NX
      Z(MX)=X(MX)+(T*MT*D(MX))
327   CØNTINUE
      CALL REQEV(NREQ,F,NX,REQ,Z)
      WRITE (6,123)(F(MZ),MZ=1,NREQ)
326   CØNTINUE
      WRITE (6,123)(SY(MY),MY=1,NREQ)
      WRITE (6,124)
124   FØRMAT (*INPUT NUMBER ØF PREFFERED PØINT*)
      READ (5,*)MN
```

```
      DØ 328 NN=1,NX
      X(NN)=X(NN)+(D(NN)*T*(MN-1))
328   CØNTINUE
      WRITE (6,125)
125   FØRMAT (*IF YØU WISH TØ TERMINATE TYPE T.  ØTHERWISE,
     *TYPE C.*)
      READ (5,130)NTER
130   FØRMAT (A1)
      IF (NTER.EQ.1HC) GØTØ 215
      WRITE (6,126)(X(MØ),MØ=1,NX)
126   FØRMAT (*ØPTIMAL X*/5F12.4)
999   STØP
      END
C
C*****THIS SUBRØUTINE EVALUATES THE RESPØNSE EQUATIØNS.
C
      SUBRØUTINE REQEV(NREQ,F,NX,REQ,X)
      DIMENSIØN F(10),REQ(10,21),X(5)
      DØ 300 JT=1,NREQ
      F(JT)=0.
      DØ 302 IS=1,NX
      F(JT)=F(JT)+(REQ(JT,IS))*(X(IS)**2)
302   CØNTINUE
      DØ 303 IA=2,NX
      F(JT)=F(JT)+(REQ(JT,IA+4))*(X(1)*X(IA))
303   CØNTINUE
      DØ 304 IB=3,NX
      F(JT)=F(JT)+(REQ(JT,IB+7))*(X(2)*X(IB))
304   CØNTINUE
      F(JT)=F(JT)+(REQ(JT,13))*(X(3)*X(4))+(REQ(JT,14))*(X(3)
     **X(5))
      F(JT)=F(JT)+(REQ(JT,15))*(X(4)*X(5))
      DØ 306 IØ=1,NX
      F(JT)=F(JT)+(REQ(JT,IØ+15)*X(IØ))
306   CØNTINUE
      F(JT)=F(JT)+REQ(JT,21)
300   CØNTINUE
      RETURN
      END
C
C*****THIS SUBRØUTINE PERFØRMS THE SUBØPTIMIZATIØN FØR
C*****INTEGER REGIØN ØF EXPERIMENTATIØN BØUNDARIES.
C
      SUBRØUTINE SINT(Y,WG,BØUN,NX)
      DIMENSIØN Y(5),WG(5),BØUN(5,2)
      DØ 320 IP=1,NX
      Y(IP)=0.
      IF (WG(IP).LT.0.) Y(IP)=BØUN(IP,1)
      IF (WG(IP).GT.0.) Y(IP)=BØUN(IP,2)
320   CØNTINUE
      RETURN
```

```
      END
C
C*****THIS SUBRØUTINE RØUTES THE PRØGRAM TØ THE PRØGRAM
C*****FØR THE SUBØPTIMIZATIØN ØF NØNLINEAR REGIØN ØF
C*****EXPERIMENTATIØN BØUNDARIES.
C
      SUBRØUTINE NLP(Y,NX), RETURNS(AAA,BBB)
      DIMENSIØN Y(5)
      WRITE (6,140)
140   FØRMAT (*IF YØU DØ NØT HAVE Y, INPUT NØ, ØTHERWISE YES*)
      READ (5,145)ITER
145   FØRMAT (A2)
      IF (ITER.EQ.2HNØ) RETURN BBB
      WRITE (6,150)
150   FØRMAT (*INPUT VALUES ØF Y*)
      READ (5,*)(Y(I),I=1,2)
      RETURN AAA
      END
```

APPENDIX E

In keeping with the hypothetical nature of Chapter IV, Equation 4.26, 4.27 and 4.28 were not actually obtained from TRADOC. An interview was conducted with Armor officers studying Operations Research at Georgia Tech. To insure commonality of independent variables for all response equations, time to fire the first round and time between rounds were treated as independent variables in the interview with trainings hours and training rounds as dependent variables.

Initially an attempt was made to fit second order equations to the training responses in the optimum region of experimentation of Equations 4.24 and 4.25. A statistically satisfactory fit was not possible in the optimum region of experimentation. A first order approximation in the optimum region of experimentation to the training curves was then fit by use of the SPSS regression program. The input to the program was:

x_1		x_2		y_3	y_4
-1	(8)	-1	(5)	48	60
1	(16)	-1	(5)	30	48
-1	(8)	1	(15)	40	54
1	(16)	1	(15)	12	19
0	(12)	0	(10)	36	48

The SPSS output is found on the next page of this Appendix, $\hat{y}_3$ on top $\hat{y}_4$ at the bottom.

The SPSS output yielded the following two response equations,

$$\hat{y}_3 = -11.5x_1 - 6.5x_2 + 33.2$$

- - - REGRESSIØN - - -

DEP. VAR... HT

FINAL STEP.

MULTIPLE R	.9760	ANØVA	DF	SUM SQUARES	MEAN SQ.	F
R SQUARE	.9525	REGRESSIØN	2.	698.000	349.000	20.057
STD DEV	4.1713	RESIDUAL	2.	34.800	17.400 SIG	.047

VARIABLE	B	S.E. B	F	SIG.	BETA	ELASTICITY
TB1	-11.500	2.086	30.402	.031	-.84964	0
TBS	-6.500	2.086	9.713	.089	-.48023	0
CØNSTANT	33.200	1.865	316.736	.003		

ALL VARIABLES ARE IN THE EQUATIØN.

DEP. VAR... LR

FINAL STEP.

MULTIPLE R	.9280	ANØVA	DF	SUM SQUARES	MEAN SQ.	F
R SQUARE	.8613	REGRESSIØN	2.	858.500	429.250	6.208
STD DEV	8.3156	RESIDUAL	2.	138.300	69.150 SIG	.139

VARIABLE	B	S.E. B	F	SIG.	BETA	ELASTICITY
TB1	-11.750	4.158	7.986	.106	-.74433	0
TBS	-8.750	4.158	4.429	.170	-.55429	0
CØNSTANT	45.800	3.719	151.673	.007		

$$\hat{y}_4 = -11.75x_1 - 8.75x_2 + 45.8.$$

After decoding the above are

$$\hat{y}_3 = -2.5556\xi_1 - 2.1667\xi_2 + 87.2009$$

$$\hat{y}_4 = -2.6111\xi_1 - 2.9167\xi_2 + 107.30015.$$

The regression F statistics are $F_{\hat{y}3} = 20.057$, significant at $\alpha = .047$, and $F_{\hat{y}4} = 6.208$, significant at $\alpha = .139$. A response equation for $\hat{y}_5$ was derived by multiplying $\hat{y}_4$ by a cost of \$90.00 per training round fired and adding a POL cost of \$10.50. Manpower costs were not included since they are fixed no matter what the personnel are doing.

BIBLIOGRAPHY

1. Agarwal, Surendra, "Optimizing Techniques for the Interactive Design of Transportation Networks Under Multiple Objectives", Northeastern University, Ph. D. Dissertation, 1973.

2. Amor, Jean-Pierre, "Acceleration of Methods of Steepest Discent Via Optical Leading", Working Paper #207, Western Management Science Institute, UCLA, August 1973.

3. Baily, R. N., Maldonado, P., McKibbins, S.W., Tarver, M. G., "Statistical Analysis and Optimization Procedure for Kraft Pulping Process", Technological Association of Pulp and Paper Industries, Vol. 52, No. 7, pp. 1272-5, July 1969.

4. Barradale, I. and Roberts, F. D. K., "Applications of Mathematical Programming to lp Approximation", Nonlinear Programming, J. Rosen, O. Mangasarian and K. Ritter (eds.), Academic Press, N. Y., 1970.

5. Bazaraa, Mokhtar S., "An Efficient Cyclic Coordinate Method for Optimizing Penalty Functions," Georgia Tech, 1973.

6. Belenson, Sheldon M. and Kapur, Kailash, "Algorithm for Solving Multicriterion Linear Programming Problems with Examples", Operations Research Quarterly, Vol. 24, No. 1, pp. 65-77, March 1973.

7. Benayoun, R., Tergny, J. and Keuneman, D., "Mathematical Programming with Multiple Objective Functions: A Solution by POP (Progressive Orientation Procedure)", Metric, IX, 2, pp. 279-99, June 1970.

8. Biles, W. E., "Response Surface Method for Experimental Optimization of Multiresponse Processes", Industrial Engineering Chemistry Process Design and Development, Vol. 14, No. 2, pp. 152-8, April 1975.

9. Bolker, H. I., "Delignification by Nitrogen Compounds", Industry Engineering Chemistry Research and Development, Vol. 4, p. 74, 1965.

10. Box, G. E. P., "The Exploration and Exploitation of Response Surfaces, Some General Considerations and Examples", Biometrics, Vol. 10, p. 16, 1954.

11. Box, G. E. P., "Some General Considerations in Process Optimization", Journal of Basic Engineering, Vol. 82, p. 113, 1960.

12. Box. G. E. P. and Hunter, J. S., "Multifactor Experimental Designs for Exploring Response Surfaces", Annals of Mathematical Statistics, Vol. 28, No. 1, p. 195, 1957.

13. Burnette, Thomas N., Jr., "A Comparison of the Applicability and Effectiveness of ANOVA With MANOVA For Use in the Operational Evaluation of Command and Control Systems", Georgia Tech Masters Thesis, May 1975.

14. Carrol, Charles W., "The Created Response Surface Technique for Optimizing Nonlinear, Restrained Systems", Operations Research, Vol. 9, pp. 169-84, 1961.

15. Charnes, A. and Cooper, W. W., Management Models and Industrial Applications of Linear Programming, John Wiley and Sons, New York, 1967.

16. Chow, W. M., "A Note on the Calculation of Certain Constrained Maxima", Technometric, Vol. 4, p. 135, 1962.

17. Courtney, James, Jr., "Differentiating Capital Appreciation and Income in Portfolio Selection (Draft)", School of Industrial Management, Georgia Tech, 1975.

18. Davies, O. L., The Design and Analysis of Industrial Experiments, Hafner Publishing Co., New York, 1956.

19. Dyer, James S., "A Time Sharing Computer Program for the Solution of the Multiple Criteria Problem", Management Science, Vol. 19, No. 12, August 1973.

20. Dyer, James, "An Empirical Investigation of a Man-Machine Interactive Approach to the Solution of the Multiple Criteria Problem", Multiple Decision Criteria Making, ed. James Cochrane, University of S. Carolina Press, 1973.

21. Dyer, James S., "Interactive Goal Programming", Management Science, Vol. 19, No. 1, p. 62, September 1972.

22. Ellis, S., Jeffreys, G., Wharton, J., "Study of the Yield and Concentration Contours of Chloramine from Raschig Synthesis of Hydrazine", Industry Engineering Chemistry Process Design and Development, Vol. 3, p. 18, 1964.

23. Fields, Timothy G., "Nonlinear Programming Techniques For the Multiple Response Problem",Georgia Tech, Master's Thesis, February 1974.

24. Fishman, George S., Concepts and Methods in Discrete Event Digital Simulations, John Wiley and Sons, New York, 1973.

25. Frank, Marguerite and Wolfe, Philip, "An Algorithm For Quadratic Programming", Naval Research Logistics Quarterly, Vol. 3, No. 1 pp. 95-109, Mar-Dec 1956.

26. Garrido, Bruce, "An Interactive Goal Programming Approach to Multi-Item Inventory Systems", Georgia Tech Master's Thesis, March 1974.

27. Geoffrion, A. M., "Vector Maximal Decomposition Programming", Working Paper No. 164, Western Management Science Institute, UCLA, September 1970.

28. Geoffrion, Arthur, M., Dyer, James S., and Feinberg, A., "An Interactive Approach for Multi-Criterion Optimization with an Application to the Operation of an Academic Department", Working Paper No. 176, Western Management Science Institute, UCLA, December 1972.

29. Geoffrion, A. M. and Hogan, W. W., "Coordination of Two-Level Organizations With Multiple Objectives", Report No. 101, Western Management Science Institute, UCLA, 1972.

30. Heller, Nelson B., and Staats, Glenn E., "Response Surface Optimization When Experimental Factors are Subject to Costs and Constraints", Technometrics, Vol. 13, No. 1, 1972.

31. Hill, W. J. and Hunter, W. G., "A Review of Response Surface Methodology: A Literature Survey", Technometrics, Vol. 8, pp. 571-90, 1966.

32. Hindeland, T. J., "QC Optimization Through Goal Programming", Quality Process, December 1972.

33. Hines, William W. and Montgomery, Douglas C., Probability and Statistics In Engineering and Management Science, Ronald Press, Co. New York, 1972.

34. Hoerl, Arthur E., "Optimum Solution of Many Variable Equations", Chemical Engineering Progress, Vol. 53, p. 69, 1959.

35. Hunter, J. S., "Determination of Optimum Operating Conditions by Experimental Methods: Part II - 1, 2, 3, Models and Methods", Industrial Quality Control, Vol. 15, pp. 6-14, 1958-9.

36. Hunter, J. S., "Statistical Methods for Determining Optimum Conditions", Trans. Tenth Annual Convention, ASQC, p. 416, 1956.

37. Ijiri, Y., Management Goals for Accounting and Control, Rand-McNally and Co., Chicago, 1965.

38. Johnsen, Erik, Studies in Multiobjective Decision Models, Studentlitterature, Leend, 1968.

39. Lee, S. M., Goal Programming For Decision Analysis, Averback, Philadelphia, 1972.

40. Lee, Lang M. and Moore, Lawrence J., "Optimizing Transportation Problems with Multiple Objectives", AIEE Transactions, Vol. 5, No. 4, pp. 333-8, December 1973.

41. Lind, E. E., Goldin, J., Hickman, J. B., "Fitting Yield and Cost Response Surfaces", Chemical Engineering Progress, Vol. 56, p. 62 1960.

42. Lind, E. E. and Young, W. R., "Con-man: A 3-D Device for the Representation of Response Surfaces", Trans. Nineteenth Annual Convention, ASQC, p. 545, 1965.

43. Michaels, S. E. and Pengilly, P. J., "Maximum Yield for Specified Cost", Applied Statistics, Vol. 12, pp. 189-93, 1963.

44. Miller, James R., Professional Decision Making: A Procedure for Evaluating Complex Alternatives, Praeger Pub., New York, 1970.

45. McNarchi, D. E., Kisiel, C. C. and Duckstein, L., "Interactive Multi-Objective Programming in Water Resources a Case Study", Water Resources Research, Vol. 9, No. 4, pp. 837-50, August 1973.

46. Montgomery, Douglas C., Talavage, Joseph J. and Mullen Cassius J., "Response Surface Approach to Improving Traffic Signal Settings in a Street Network", Transportation Research, Vol. 6, No. 1, pp. 69-80, March 1972.

47. Myers, Raymond H., Response Surface Methodology, Allyn and Bacon, Inc., Boston, 1971.

48. Myers, Raymond and Carter, Walter H., Jr., "Response Surface Techniques for Dual Response Systems", Technometrics, Vol. 15, No. 2, 1973.

49. Nie, Norman H., et al., Statistical Package for the Social Sciences, McGraw-Hill Co., New York, 1975.

50. Operations Research Associates, Operational Test Methodology Guide, Vol. II, Techniques and Guidelines, January 1974.

51. Remmers, E. G. and Dunn, G. G., "Process Improvement of a Fermentation Product", Industry Engineering Chemistry, Vol. 53, p. 743, 1961.

52. Ruefli, T. W., "A Generalized Goal Decomposition Model", Management Science, Vol. 17, pp. 508-18, April 1971.

53. Schrage, R. W., "Optimizing a Catalytic Cracking Operation by the Method of Steepest Ascent", Operations Research, Vol. 6, pp. 498-515, 1958.

54. Seinfeld, John H. and McBride, Warren L., "Optimization With Multiple Performance Criteria", Industrial and Engineering Chemistry Process Design and Development, Vol. 9, No. 1, pp. 53-57, January 1970.

55. Smith, H. and Rose, A., "Subjective Responses in Process Investigation", Industry Engineering Chemistry, Vol. 55, p. 25, 1963.

56. Taraman, Khalil S., Lambert, Brian K., "Application of Response Surface Methology to the Selection of Machining Variables", AIIE Transactions, Vol. 4, No. 2, pp. 111-15, June 1972.

57. Umland, A. W. and Smith, W. N., "The Use of LaGrange Multipliers With Response Surfaces", Technometrics, Vol. 1, pp. 289-92, 1959.

58. Underwood, W., "Experimental Methods for Designing Extrusion Screws", Chemical Engineering Progress, Vol. 58, No. 1, p. 59, 1962.

59. U. S. Army Training and Doctrine Command, "Analyzing Firing Effectiveness (Draft)", April 1975.

60. U. S. Department of the Army, Basic Policies for Systems Acquisition by the Department of the Army, AR 1000-1. Washington: U.S. Government Printing Office, November 1974.

61. U. S. Department of the Army, Organization and Functions, U. S. Army Operational Test and Evaluation Agency, AR 10-4, Washington: U. S. Government Printing Office, December 1974.

62. U. S. Department of the Army, Research and Development and Force Development Testing, AR 71-3, Washington: U. S. Government Printing Office, November 1974.

63. U. S. Department of Defense, Acquisition of Major Defense Systems, DOD Directive 5000.1, Washington: U. S. Government Printing Office September 1974.

64. U. S. Department of Defense, The Decision Coordinating Paper (DCP) and the Defense Systems Acquisition Review Council (DSARC), DOD Directive 5000.2, Washington: U. S. Government Printing Office, February 1974.

65. Vemuri, V., "Multiple Objective Optimization in Water Resource Systems", Water Resources Quarterly, Vol. 10, No. 1, pp. 44-8, 1974.

66. Williams, R. A. and McGilchrist, C. A., "Experimental Study of Drill Life", International Journal of Production Research, Vol. 10, No. 2, pp. 175-91, April 1972.

67. Wolf, D., "Convergence Theory in Nonlinear Programming", Integer and Nonlinear Programming, ed. J. Abadie, North-Holland Publishing Co., Amsterdam, 1970.

68. Wu, S. M., "Tool Life Testing by Response Surface Methodology", Part 1, ASME Transactions, Ser B., Journal Engineering Industry, p. 105, 1964a.

69. Zangwill, Willard I., Nonlinear Programming: A Unified Approach, Prentice Hall, New Jersey, 1969.